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# PROBLEMS

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# ASTRONOMY, SURVEYING, AND NAVIGATION:

## WITH THEIR SOLUTIONS.

By H. W. JEANS, F.R.A.S.

ROYAL NAVAL COLLEGE;
AN EXAMINER OF MERCHANT OFFICERS IN NAUTICAL ASTRONOMY, ETC.;
AND FORMERLY MATHEMATICAL MASTER IN THE ROYAL
MILITARY ACADEMY, WOOLWICH.

New Edition.

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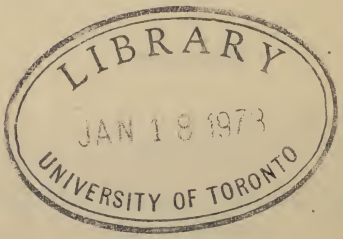


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# PREFACE.

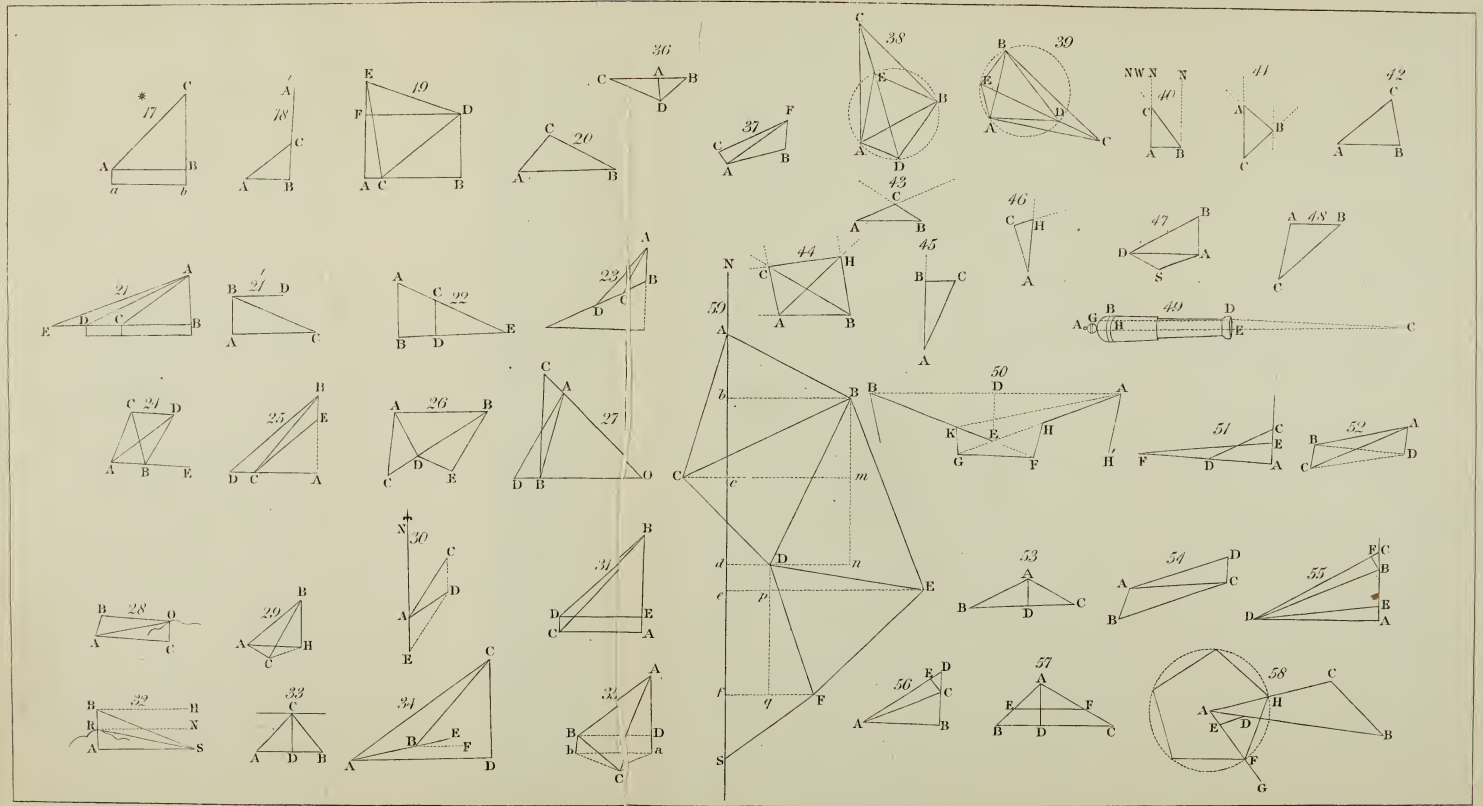
THE following pages have been drawn up chiefly with the object of facilitating *self-instruction* in the principles of Navigation and of Nautical Astronomy. Designed for the use of all parties interested in navigating ships, they are especially adapted for those who have to rely on their own exertions for the knowledge which they may require.

The theory of Navigation is treated concisely and familiarly. When that is learned in connexion with the rules derived from it, the subject assumes a more attractive form, and retains firmer hold on the mind.

By means of a few practical rules, the attention of seamen is drawn to the importance and method of correcting the compass for the local deviation arising from the effects of the induced magnetism of the iron on board. The examples under each rule have been added at the suggestion of several naval instructors, who have thought that a series of easy examples would tend to familiarise

their pupils with the rules; and in order still further to meet their wishes, additional copies of this part of the book have been struck off and bound in a separate volume, with a view to its adoption as a class-book.

*Sept.* 10, 1849.



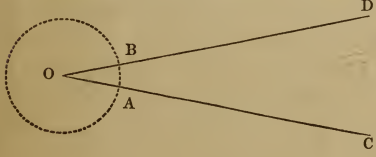
* Figures 1 to 16 are inserted in the pages of Part 1.

# PROBLEMS

IN

# ASTRONOMY, SURVEYING, AND NAVIGATION.

When a line joining any two points in space is accessible throughout its whole extent, it may in general be measured by the successive application of some line of a known length; but when it is inaccessible, or cannot be directly measured, we may obtain its length by considering it the *side of a triangle*, if we already know, or can find by observation, a sufficient number of parts of the triangle to enable us to apply one or more of the rules of Plane Trigonometry. It is thus that the mensuration of inaccessible lines of any length is found by means of that of accessible lines and angles.




The angle DOC, contained by lines drawn from a point, O, to two remote objects, C and D, may be measured by placing a circle in the plane passing through the two objects, and having its centre at the angular point, O: the straight edge of a ruler being then placed on the circle so as to pass through its center, and, by means of

sights placed over it, directed first to one object and then to the other, the arc, AB, of the circumference between the two positions of the ruler can be found; this is manifestly the measure of the angle O.

The principal instruments for measuring angles are the theodolite and sextant. A Theodolite is the most convenient instrument for measuring horizontal and vertical angles: it is composed of two circles having their planes perpendicular to each other. When the instrument is used, one of the circles is placed in a horizontal plane, by means of levels; on this circle horizontal angles are measured; on the other are measured vertical angles, whether of elevation or depression (that is, whether the object is above or below the horizontal line). A Sextant is employed to measure angles contained in any plane whatever. It is more suited for observing angular distances of heavenly bodies than the theodolite; but the latter is better adapted for Surveying than the former, since it determines the horizontal angles at once; but those observed with the sextant must, when out of the plane of the horizon, be reduced to that plane by calculation, to suit them to the purposes of the survey. In order to apply the rules of Spherical Trigonometry to the solution of astronomical problems, such as finding the latitude or hour-angle of a heavenly body, it is necessary to conceive the required arc or angle to be a part of a spherical triangle, two other parts of which are previously known by means of the Nautical Almanac or otherwise; a third part is usually found by observing with the sextant the altitude of the heavenly body above the horizon, or its angular distance from some other body, according to the

nature of the problem; we thus obtain sufficient data for computing the value of the part required.

We will not stop to give particular descriptions of the theodolite and sextant: their construction will be best learned by a careful study of the instruments themselves; we shall therefore suppose the manner of adjusting and applying them to practice is known, and proceed to give a collection of problems which they enable us to solve.

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## SECTION I.

### PROBLEMS IN SURVEYING, ETC.

[1]. (Fig. 17.) On the opposite bank of a river to that on which I stood, is a tower known to be 216 feet high; with a pocket sextant I ascertained the angle between a horizontal line drawn from my eye (supposed to be 5 feet above the ground) and its top to be $47°\,56'$; required the distance across the river, from the place where I stood, to the bottom of the tower.

*Ans.* 190·4 feet.

[2]. (Fig. 53.) BDC is a straight line to which AD is perpendicular; AD is 100 feet high, and subtends an angle at $B = 36°\,48'$, and at C an angle $= 54°\,30'$; find the length of the line BC. *Ans.* 205 feet.

[3]. (Fig. 17.) A field is in the form of a right-angled triangle, whose base is 200 feet, and the angle at the base is 67°; how long will a man be walking round it at the rate of 4 miles an hour?

*Ans.* 3 min. 21·6 sec.

[4]. (Fig. 53.) C and B are two places 100 feet apart, and A is a point equally distant from C and B; what must be the distance of the point A that the angle BAC may be 150°? *Ans.* 51·77 feet.

[5]. (Fig. 18.) A May-pole being broken off by the wind, its top struck the ground at 15 feet distance from the foot of the pole; required the height of the whole May-pole, supposing the length of the broken piece to be 39 feet. *Ans.* 75 feet.

[6]. (Fig. 19.) A ladder 36 feet long may be so placed that it shall reach a window 30·7 feet from the ground on one side of the street, and by only turning it over, without moving the foot out of its place, it will reach another window 18·9 feet high on the other side. What is the breadth of the street, and the angle of elevation of the first window above the second?

*Ans.* Breadth of street 49·44 feet.
Angle of elevation 13° 25′.

[7]. (Fig. 17.) From the bottom of a tower, a distance AB = 50 yards is measured on a horizontal plane, and at A, the angle BAC is found = 25° 17′; required the height of the tower BC.

*Ans.* 23·6 yards.

2*d*, When AB=12¼ and BAC=45°. ,, 12¼ ,,

[8]. (Fig. 20.) To determine the distance of a ship at anchor at C, I measured a straight line AB=1000 yards on the shore, and observed the angles CAB=32°10′ and CBA=83° 18′; required the distance of the ship from A. *Ans.* 1100·1 yards.

2*d*, When AB=2·5 miles, CAB=80°, CBA=50°.

*Ans.* 2·5 miles.

[9]. (Fig. 20.) Two ships sailing in company,

in order to determine nearly their distance from an object, C, on the shore, are separated from each other two nautical miles, AB; the angle is then observed from each ship between the object and the other ship; at A the angle is 85° 10′, at B it is 82° 45′; required the distance of each ship from the object.

*Ans.* 9·478, and 9·52 miles.

[10]. The base of an isosceles triangle is 120 feet, and the vertical angle 47°; find the sides.

*Ans.* Each side 150·5 feet.

[11]. (Fig. 17.) The two sides, AB and BC, of the right-angled triangle ABC are 18 and 24; find the length of the perpendicular drawn from the right angle upon the hypothenuse. *Ans.* 14·4.

[12]. (Fig. 21.) To determine the height, AB, of a tower inaccessible at the base, two stations, C and D, are chosen in a horizontal plane, so that a plane passing through the three points A, C, and D, is perpendicular to the horizon; the distance CD=100 yards, and the angle ACB=46° 15′, and BDA=31° 20′; required the height AB. *Ans.* 145·9 yards.

2*d*, When CD=300 yards, ACB=58°, and BDA=32°.

*Ans.* 307·4 yards.

[13]. (Fig. 21.) From the decks of two ships at D and C, 880 yards asunder, the angle of elevation of a cloud at A on the same point of the compass from each is observed; at D the angle is 35°, at C it is 64°; required the height of the cloud above the surface of the sea, the deck of each ship being supposed to be elevated above it 21 feet. *Ans.* 942·6 yards.

[14.] A tower subtended 39° to an observer stationed 200 feet from the base; find the height, and also

the angle it will subtend to an observer at 350 feet from the base. *Ans.* Height, 162 feet; angle 24° 50′.

[15] (Fig. 20.) To determine nearly the distance between two ships at sea, I carefully observed the interval of time between the flash and report of a gun from each, and measured the angle which the two ships subtended. The intervals were 4 seconds and 6 seconds, and the angle observed = 48° 42′; required the distance of the ships (the velocity of sound being supposed to be 1142 feet in a second). *Ans.* Distance 5147·9 feet.

2*d*, When the intervals are 10 seconds and 5 seconds, and observed angle 60°. *Ans.* Dist. 9889·8 feet.

[16]. (Fig. 21′.) From B, the top of a ship's mast, which was 80 feet above the water, the angle of depression* of another ship's hull at C upon the water was 20°; required the distance of the ships.

*Ans.* 219·8 feet.

2*d*, If the mast = 143 feet, and the angle of depression = 35°. *Ans.* 204·2 feet.

[17]. There are two monuments whose heights are 100 feet and 50 feet respectively: I observed, with a sextant, that the line joining their tops when produced made with the horizontal plane an angle = 37°; required their distance apart. *Ans.* 66·4 feet.

[18]. (Fig. 23.) Wishing to determine the height

* The angle between a horizontal line passing through the eye of the observer and a line drawn to the object is called the *angle of elevation* when the object is above the horizontal line, and the *angle of depression* when the object is below it; thus in fig. 21′, BCA is the angle of elevation of B above the horizontal line AC, and DBC is the angle of depression of C below the horizontal line BD.

of an obelisk standing on the top of a declivity, I measured from its base a distance of 40 feet, and then observed the angle formed by this line and a line drawn to the top to be = 41°. Going on in the same direction 60 feet farther, the angle formed by the declivity with a line drawn to the top was = 23° 45′. Required the height of the obelisk. *Ans.* Height, 57·62 feet.

[19]. (Fig. 24.) To determine the distance between two inaccessible objects, C and D, as two ships at anchor, a base, AB, on the same plane as the objects was measured and found to be 670 yards; the following angles were also observed at the extremities of the base: at A, BAD = 40° 16′, BAC = 97° 56′; at B, ABC=42° 22′, and ABD=113° 29′; required CD.

*Ans.* CD=1174·4 yards.

[20]. (Fig. 24.) To determine nearly the distance of two redoubts, C and D, by which the entrance into a harbour is defended, a boat is placed at A with its head towards a distant object seen at E, and the angles CAD=22° 17′, and DAE=48° 1′ were observed. The boat is then moved to B, a distance of 1000 yards, directly towards E, and the angles CBD=53° 15′, and DBE=75° 43′ are observed; required the distance CD.

*Ans.* Distance, 1290 yards.

[21]. A right-angled triangle rests on its hypothenuse, the length of which is 100 feet; one of the angles is 36° 40′, and the inclination of the triangle to the horizon is 60°; find the height of the right angle above the ground. *Ans.* 41·48 feet.

[22]. (Fig. 25.) To determine the height of an object, EB, on the top of an inaccessible hill, I took the angle of elevation, ACE, of the top of the hill = 40°,

and also that of the top of the object ACB=51°. Going then 100 yards in a direct line from the object and upon a horizontal plane, I found the angle of elevation of the top of the object ADB=33° 45′; required the height of the object. *Ans.* Height, 46·67 yards.

[23]. (Fig. 26.) Wanting to know the distance between two objects, A and B, which could only be seen from a particular place, D, I set up two staffs, at C and E, and took the angles ADC=89°, ADB=72° 30′, and BDE=54° 30′. I then measured DE and DC, each = 200 yards, and took the angles BED = 88° 30′ and DCA=50° 30′; required the distance AB.

*Ans.* 345·5 yards.

[24]. (Fig. 27.) To determine my distance from an inaccessible object at O, *without observing any angles*, I measured a straight line, AB=500 yards, from each extremity of which I could see O. I then measured from A and B, in a direct line from O, AC and BD, each = 100 yards; finally, I measured the diagonals AD and BC, the former was = 550 yards, the latter = 560 yards; required the distance of the object from A and B.

*Ans.* AO=536·2, BO=500·47 yards.

[25]. (Fig. 28.) Wanting to know the distance, AC, of a hill from the station A, and also its height, OC, I measured a base, AB=298 yards, on ground nearly level, and at A and B, observed with a sextant the angles BAO=42° 17′ and ABO=79° 29′; and at A, the angle of elevation OAC=4° 51′; required the distance AC and height OC.

*Ans.* AC=344·6, OC=29·2 yards.

[26]. (Fig 29.) Find how much the point B is elevated above a point H, from the following data: hav-

ing observed the angle of elevation BAH=4° 38′ and measured AC=193 yards to a point C, at C the angle ACB=76° 32′, and at A the angle BAC=45° 4′.

*Ans.* BH=17·8 yards.

[27]. (Fig. 29.) The angle of elevation of a tower 100 feet high, due north of an observer, was 50°; what will be its angle of elevation after walking due east 300 feet? *Ans.* 17° 47′ 45″.

[28]. (Fig 30.) The elevation of a balloon was observed to be 20° bearing N.E., and by another observer 4000 yards due south of the former it was N.b.E.; required its height. *Ans.* 511·3 yards.

[29]. (Fig. 31.) From a window which seemed to be on a level with the bottom of a steeple, I observed the angle ACB of elevation of the top of the steeple = 40°; from another window of the same house, 18 feet directly above the former, the angle EDB of elevation was = 37° 30′; required the height and distance of the steeple.

*Ans.* Height, 210·4; distance, 250·8 feet.

[30]. (Fig. 32.) At B, the top of a castle which stood on a hill near the sea-shore, the angle of depression, HBS, of a ship at anchor was 4° 52′, and at R, the bottom of the castle, its depression, NRS, was 4° 2′; required the height of the top of the building above the level of the sea, supposing the castle itself to be 54 feet high; required also the horizontal distance of the vessel.

*Ans.* Height, 314·2; distance, 3690·3 feet.

[31]. (Fig. 33.) Wishing to know the breadth of a river, I measured a base of 500 yards in a straight line close to one side of it, and at each extremity of the base I observed the angles subtended by the other end and a

tree standing on the opposite bank to be 53° and 79°12′; required the breadth of the river.

*Ans.* Breadth, 529·5 yards.

2*d*, If base = 108·3 and angles 43° 12′ and 60° 35′.

*Ans.* Breadth, 66·49.

[32]. (Fig. 34.) A base of 340 feet was measured on a sloping side of a hill, in a vertical plane passing through its summit. It made an angle of 10° 5′ with the horizontal plane; at the higher end of the base, the elevation of the hill was 46° 15′, and at the other end it was 40° 10′. The height of the eye at each observation was 5 feet; find the height of the hill above the lower end of the base. *Ans.* 1226 feet.

[33]. (Fig. 35.) The distance between two stations B and C on a declivity is 220 yards. At B, the *horizontal* angle, C*ba*, between C and an object A on the top of a hill was found, by a *theodolite*, to be 70° 15′, and at C, the horizontal angle *b*C*a*, between B and A, was 62° 33′; the vertical angles AC*a*=32° 12′ and BC*b*=8° 32′; find the horizontal distances of the object A from C and B, namely, C*a* and *ba*, and also the heights A*a* and AD of the object above C and B.

*Ans.* Hor. dist. of A from C = C*a* = 279·1 yards.
,, ,, B = *ba* = 263·1 ,,
perp. height of A above C = A*a* = 175·7 ,,
,, ,, B = AD = 143·1 ,,

[34]. (Fig. 35.) Suppose, as in the last problem, that the direct distance between the stations B and C is 220 yards, also that at C the angle BC*b* (the elevation of B) = 8° 32′ and AC*a* (the elevation of A) = 32° 12′; but suppose now that a *sextant* is used in measuring the oblique angles, and that ABC = 77° 8′ and ACB = 62° 18′.

Find, as before, the *horizontal* distances between A and the stations B, C, and the heights of A above B and C.

*Ans.* See last problem.

[35]. (Fig. 36.) Wanting to know the distance of two objects, A and B, from each other, and from another object, D, all in the same plane, on BA, produced on the side of A, a point C was taken, and CD being measured was found to be 549·4 yards, and the angle C=57°. At D the angle CDA was observed to be 14°, and the angle BDA = 41° 30′; required the distance of A, B, and D from each other.

*Ans.* AB=349·52, AD=487·27, and BD=498·7 yards.

[36]. (Fig. 37.) Wishing to find the distance of a battery at B, from a fort at F, which cannot be seen from the battery in consequence of the ground between B and F being covered with wood, &c., I measured distances, BA and AC, to points A and C, where both the fort and battery were visible, the former being 2000 yards and the latter 3000; and observed the angles BAF=34° 10′, FAC=74° 42′, and FCA=80° 10′. From these data it is required to find the distance of the fort from the battery. *Ans.* 5422 yards.

[37]. (Fig. 38.) Coming from sea, at the point D, I observed two headlands A and B, and inland at C a steeple which appeared between the headlands: I found from a map that the headlands were 5·35 miles from each other; that the distance from A to the steeple was 2·8 miles, and from B to the steeple 3·47 miles. I observed with a sextant the angle ADC=12° 15′ and BDC=15° 30′; required my distance from each of the three objects.

*Ans.* AD=11·26, CD=12·46, BD=11·03 miles.

[38]. (Fig. 39.) Required the distance of the three objects A, B, and C from the point D, situated *within* the triangle, from the following data: AB=267, AC=346, BC=209, angles ADC=128° 40′, and ADB=91° 20′. *Ans.* 189·33, 178·85, 104·05.

The following twelve problems require a knowledge of the compass.

[39]. (Fig. 40.) A headland C bore due north of a ship at A; after sailing 10 miles due east to B, the headland bore N.W.; required the distance of the headland at each observation.

*Ans.* 10 and 14·14 miles.

[40]. (Fig. 41.) A fort A bore from a ship C due north, by compass, and after sailing N.E. by compass $14\frac{1}{2}$ miles to B, the fort bore N.W.; find the distance of the fort at both observations.

*Ans.* 20·5 and 14·5 miles.

[41]. (Fig. 42.) A boat is placed at A due west of a ship at B; after sailing N.W. 10 miles to C, the boat bears S.S.W.; required the distance of the boat from the ship at the first observation. *Ans.* 10 miles.

[42]. (Fig. 43.) Sailing along a coast, a headland C was observed to bear N.E.b.N.; having run E.b.N. 15 miles to B, the headland bore W.N.W.; find the distance from the headland at each observation.

*Ans.* 8·496, 10·81 miles.

[43]. A and B are two points lying north and south 50 yards apart; what must be the distance of a third point C from each, that it may bear N.b.W. from B, and W.b.S. from A?

*Ans.* 9·75 and 49·04 yards.

[44]. (Fig. 44.) A cape C was observed to bear from us N. W., and another headland H to bear N. N. E. $\frac{1}{2}$ E.; standing away E. N. E. $\frac{1}{2}$ E. 23 miles to B, we found the first bore from us W. N. W., and the second N. b. W. $\frac{1}{2}$ W.; required the distance and bearing of the cape from the headland.

*Ans.* W. $\frac{1}{4}$ S.; 42·33 miles.

[45]. A ship sailing N. W., two islands appeared in sight, one bearing W. N. W., the other N. from the ship, and when the ship had sailed 6 miles farther, the first bore W. b. S., and the other N. E.; required their bearing and distance from each other.

*Ans.* S. 58° 40′ W., distance 9·71 miles.

[46]. From two stations on the deck of a ship 100 feet apart, the bearings of an object on shore were N.E. and N.N.E., and the ship's head was N.b.W.; find the distances of the object from each station.

*Ans.* 145·2 and 217·3 feet.

[47]. (Fig. 45.) A church C bears from a battery B, E. N. E. 960 yards; how must the church bear from a ship at sea, supposing her to run in until the battery is north 2000 yards? *Ans.* N. 20° 32′ E.

[48]. (Fig. 46.) A cape C bears from a headland H, W. $\frac{1}{4}$ S. 4·23 miles; how must the cape bear from a ship which runs in towards the headland on a N. b. W. $\frac{1}{2}$ W. course, until the headland is 2·3 miles distant from the ship? *Ans.* W. N. W.

[49]. (Fig. 47.) A ship S was 2640 yards due south of a lighthouse AB, and after sailing N. W. b. N. 800 yards to D, its angle of elevation BDA was 5° 25′; required its height. *Ans.* 192 yards.

[50]. (Fig. 48.) The bearings of two objects A

and B in the same latitude from a ship at C are N.N.E. and N.E.b.E., and the distance from A is 10 miles; required the distance from B. *Ans.* 16·63 miles.

[51]. (Fig. 49.) Find the angle which the *line of metal* makes with the axis produced in a piece whose dimensions are known.

EXAMPLE.

Diameter of breech = 12·44 inches; of muzzle = 9·84 inches; length of gun = $77\frac{1}{4}$ inches. *Ans.* 0° 57′ 51″.

*Calculations of a front of Fortification.*

[52]. (Fig. 50.) Let AB be the exterior side of a regular pentagon, and suppose the side to be 350 yards, the perpendicular DE 50 yards, the face AH of the bastion 100 yards, and the line of defence AG equal to AK. As the length and position of the other lines depend on these values, it is required to find the magnitude and position of the following parts:

1. Angle of the tenaille AEB. *Ans.* 148° 6′ 30″.
2. Line of Defence AK or AG. ,, 255·33 yards.
3. Flanked angle $HAH_1$. ,, 76° 6′ 30″.
4. Curtain FG. ,, 141·02 yards.
5. Angle of the shoulder AHF. ,, 117° 0′.
6. Length of flank FH. ,, 43·5 yards.
7. Flanked angle GFH. ,, 101° 3′.

[53*a*]. What angle will a tower subtend at a place whose distance is equal to six times its height? and where must an observer station himself that the angle of elevation may be double of the former?

*Ans.* 9° 28′; 2·9 times the height.

[54*a*]. (Fig. 51.) A maypole was broken by the wind, and its top struck the ground 20 feet from its

base, and being again fixed was broken a second time 5 feet lower, and its top extended 10 feet farther; required its height. *Ans.* 50 feet.

[55*a*]. (Fig. 52.) The summit A of a hill bore due east of a spectator at B, and E. N. E. of a spectator at a point C due south of B; the elevation of the point A at B was 20°; required its elevation at C.

*Ans.* 18° 36′.

[56*a*]. (Fig. 53.) If the base of an oblique-angled triangle be 40, and the other two sides 20 and 30, what is the length of the perpendicular from the vertical angle? *Ans.* 14·523.

[57]. (Fig. 54.) The elevation of a spire DC at one station A was 23° 50′ 15″, and the horizontal angle at this station, between the spire and another station B, was 93° 4′ 20″; the horizontal angle at B was 54° 28′ 30″, and the distance AB between the two stations was 416 feet; required the height of the spire.

*Ans.* 278·7 feet.

[58*a*]. (Fig. 52.) An observer finds the angle of elevation of a tower at a point B to be 23° 18′; after walking from B 300 feet in a direction at right angles to the line joining B with the foot of the tower, the angle of elevation was 21° 16′; required the height of the tower, and its distance from B.

*Ans.* Height, 272·7 feet; dist. 633·4.

[59]. A pole is inclined at an angle of 80° to the horizontal plane, and when standing 100 feet from it in the direction in which it is inclined, the angle of elevation of the top is 54°; required its length.

*Ans.* 112·5 feet.

[60*a*]. (Fig. 55.) On the bank of a river stands a

column 200 feet high, on which is a statue 50 feet high, and to an observer on the opposite bank the statue subtended an equal angle with a man 6 feet high standing at the base of the column; required the width of the river. *Ans.* 82·5 feet.

[61*a*]. (Fig. 24.) The distance between two objects, C and D, is known to be 6594 yards; on one side of the line CD there are two stations, A and B, at which angles are taken. The angle CAD=85° 46′, DAB=23° 56′, CBD=68° 2′, and CBA=31° 48′. From these observations it is required to find the distance between the stations A and B. *Ans.* AB=4694 yards.

[62*a*]. (Fig. 56.) A flag-staff, 12 feet high, on the top of a tower, subtended an angle of 48′ 20″ to an observer at the distance of 100 yards from the foot of the tower; required the height of the tower.

*Ans.* 401·4 feet.

[63]. Find the area of a triangle whose base is 40 feet, and perpendicular 30 feet. *Ans.* $66\frac{2}{3}$ square yards.

[64*a*]. If from a right-angled triangle whose base is 12 and perpendicular 16 feet, a line be drawn parallel to the perpendicular, cutting off a triangle whose area is 24 feet, required the sides of the triangle.

*Ans.* 6, 8, 10 feet.

[65*a*]. Required the side of an equilateral triangle, the area of which is 180 square yards.

*Ans.* 20·389 yards.

[66*a*]. Given the base of a triangle equal to 476·25 yards, and the angles at the base 27° 10′ 15″ and 35° 10′ 15″, to find the area. *Ans.* 33680.

[67*a*]. The area of a triangle is 6, and two of its sides are 3 and 5; find the third side. *Ans.* 4 or $\sqrt{52}$.

[68*a*]. (Fig. 57.) The straight line EF is drawn parallel to the base of the triangle ABC whose altitude is 10 feet; find the distance of EF from the base BC, so that it may divide the triangle into two equal parts.

*Ans.* Distance from base, 2·929 feet.

[69*a*]. When a parish was inclosed, the allotment of one of the proprietors consisted of two pieces of ground, one of which was in the form of a right-angled triangle; the other was a rectangle, one of the sides of which was equal to the hypothenuse of the triangle, the other to half the greater side; but wishing to have his land in one piece, he exchanged his allotments for a square piece of ground of equal area, one side of which equalled the greater of the sides of the triangle which contained the right angle. By the exchange, he found he had saved 55 yards of paling; what are the area of the triangle and rectangle?

*Ans.* Triangle, 181·5; rectangle, 302·5 yards.

[70*a*]. The area of a right-angled triangle whose sides are in arithmetical progression $=216$; determine the sides. *Ans.* 18, 24, 30.

[71*a*]. What is the side of that equilateral triangle whose area cost as much paving, at 8*d.* per foot, as the pallisading the three sides did at 7*s.* a foot?

*Ans.* 72·74 feet.

[72*a*]. Given a side, $a$, of a regular polygon of $n$ sides, to find the area.

*Ans.* $\text{Area} = \frac{na^2}{4} \cot. \frac{180°}{n}$

EXAMPLES.

1. An eight-sided polygon, or octagon, whose side $=$ 16 yards. *Ans.* Area $=1236·1$.

2. A ten-sided polygon, or decagon, whose side $=20\frac{1}{2}$ yards. *Ans.* Area $=3233{\cdot}5$.

[73*a*]. Given the area A of a regular polygon of $n$ sides, to find a side $a$.

*Ans.* $a=\sqrt{\frac{4A}{n}\tan.\frac{180^\circ}{n}}$

EXAMPLES.

1. The area of a regular octagon is $1236{\cdot}1$ square yards; find a side. *Ans.* Side 16 yards.

2. The area of a decagon is $3233{\cdot}5$ square yards; find a side. *Ans.* Side $20{\cdot}5$ yards.

[74*a*]. (Fig. 58.) To make a regular polygon of $n$ sides equal to a given triangle, ABC. Take $AD=\frac{AB}{n}$; draw AG, making angle $CAG=\frac{360^\circ}{n}$; draw DE parallel to AC; and take AF = a mean proportional to AC and AE; then AF is the radius of a circle that will contain the required polygon; required a proof.

[75]. A ship sailing on a S.S.W. course, bore from me due south, and the angle subtended by the head and stern was $20'\,15''$, and her length was known to be 160 feet; required her distance. *Ans.* $1{\cdot}94$ miles.

[76*a*]. A ship sailed S.$x^\circ$W., and met another ship which had sailed N.$(x+10^\circ)$W. from the same meridian; the distances sailed were as 3 : 2, and their distance from the meridian left was 100 miles; required the difference of latitude. *Ans.* $479{\cdot}8$ miles.

[77*a*]. How far may the surface of the sea be seen by a man standing 6 feet above it, the radius of the earth being 4000 miles? *Ans.* 5307 yards.

[78*a*]. From the top of a mountain 3 miles high, the true depression of the horizon was found to be 2° 13′ 27″; required the diameter of the earth, supposing it to be a sphere. *Ans.* 7952 miles.

[79*a*]. If at the top of a mountain the true depression of the horizon of the sea is found to be 1°31′, what is the height of the mountain, supposing the earth to be a sphere whose diameter is 8000 miles?

*Ans.* 1·402 miles.

[80*a*]. From a station on the side of a river whose banks are parallel, and width 560 yards, the angle formed by two objects on the opposite bank was 35° 10′, and their distance from each other 400 yards; required their distance from the station.

*Ans.* 560·1 and 694·3 yards.

[81*a*]. Walking along a road, I observed the elevation of a tower, AB, to be 20°, and the angular distance of its top from an object in the road to be 30°, the nearest distance of the tower from the road was 200 feet; required its height.

*Ans.* 187·5 feet.

[82*a*]. One angle of a triangle is 129° 34′, and the two sides about that angle are to each other in the proportion of 4 to 7; required the other two angles.

*Ans.* 32° 41′ 7″ and 17° 44′ 53″.

[83*a*]. The three sides of a plane triangle = 6, and the angles are to each other as 1, 2, 3; find the sides.

*Ans.* 1·268, 2·536, 2·196.

[84*a*]. The perimeter of a triangle is equal to 100 yards, and the angles are to each other in the proportion of 1, 2, and 4: it is required to find the sides of the triangle. *Ans.* 19·8, 35·69, 44·51 yards.

[85*a*]. The perimeter of a right-angled triangle = 24 yards, and one of the angles = 30°; find the sides.

*Ans.* 5·072, 8·784, 10·144 yards.

[86*a*]. In a plane triangle, ABC, given A=80°, and $a$=400; required the other sides, their sum being 600.

*Ans.* 369·25, 230·75.

[87*a*]. In a plane triangle, ABC, given A = 60°, $c$=400, and the sum of the other two sides, $a$ and $b$,=600; required the sides $a$ and $b$.

*Ans.* 250 and 350.

[88*a*]. The perimeter of a right-angled triangle is 24 feet, and its base is 8 feet; find the other sides.

*Ans.* 6 and 10 feet.

[89*a*]. At 80 feet distance from a steeple, the angle made by a line drawn from its top to the place was double to that made by a line drawn from the top to a point 250 feet from the steeple on the same level; required the height of the steeple. *Ans.* 150 feet.

[90*a*]. Given the base $a$, the vertical angle A, and the sum of the sides of the plane triangle ABC=$b$, to find the sides $x$ and $y$.

*Ans.* $x+y=b$, and $xy=\frac{1}{4}(b+a).(b-a)\ \text{sec.}^2\frac{A}{2}$

from which equations $x$ and $y$ may be found.

[91*a*]. Given the base $a$, the vertical angle A, and the difference of the sides of a plane triangle = $d$, to find the sides $x$ and $y$.

*Ans.* $x-y=d$, and $xy=\frac{1}{4}(a+d).(a-d)\ \text{cosec.}^2\frac{A}{2}$

[92*a*]. Given the base $a$, the difference of the angles $X$ and $Y$ at the base = $d$, and the sum of the two other sides of the plane triangle = $b$, to solve the triangle.

*Ans.* Cos.$\frac{1}{2}(X+Y)=\frac{a}{b}$cos.$\frac{1}{2}d$, which determines the sum of the unknown angles, and thence, with their difference already known, the angles $X$ and $Y$.

[93*a*]. Given the base $a$, the difference of the angles $X$ and $Y$ at the base = D, and the difference of the sides of a plane triangle = $d$, to solve the triangle.

*Ans.* Sin.$\frac{1}{2}(X+Y)=\frac{a}{d}$sin.$\frac{1}{2}$D, and $X-Y$ is already known.

[94*a*]. Given the base of a triangle $b$, and one of the angles at the base A, and the difference of the other sides = $d$, to solve the triangle.

*Ans.* Tan.$\frac{1}{2}$C$=\frac{b-d}{b+d}$tan.$\frac{1}{2}$A, from which the angle C is found, and thence the other parts of the triangle.

[95*a*]. Given the base of a plane triangle $b$, one of the angles at the base A, and the sum of the other sides = $m$, to solve the triangle.

*Ans.* Tan.$\frac{1}{2}$C$=\frac{m-b}{m+b}$cot.$\frac{1}{2}$A.

[96*a*]. Given the angles and the perimeter of a plane triangle, to find the sides.

EXAMPLE.

Perimeter = 100 yards, A = 102°51′30″, B = 25°42′45″, and C = 51°25′45″.

*Ans.* $a$ = 44·51, $b$ = 19·8, $c$ = 35·69 yards.

[97*a*]. Given the distances between three stations in a straight line with an object standing upon a horizontal plane; and the angles at the points E, D, C, = $\theta$, $90-\theta$, and $2\theta$ in order, $\theta$ being unknown, to find its height.

EXAMPLE.

Let ED = 20, and DC = 20. *Ans.* 38·73.

[98*a*]. Investigate analytical expressions for calculating the distance of a station from each of three points, having given the distances of the points from each other, and the angles which they subtend at the station. For numerical examples, see Probb. 37, 38.

[99*a*]. If in the three edges which meet at one angle of a cube, three points, A, B, C, be taken at distances $a$, $b$, $c$, from the angle respectively, the area of the triangle ABC, formed by joining the three points with each other, $= \frac{1}{2}\sqrt{a^2b^2+a^2c^2+b^2c^2}$.

[100*a*]. On the sides of an equilateral triangle three squares are described. Shew that the area of the triangle formed by joining the centers of these squares = area equilateral triangle $\times (1+\frac{1}{2}\sqrt{3})$.

[101*a*]. A fleet of steam-vessels, under the command of an admiral, is steaming due east, 6 miles an hour; a part of the fleet under a vice-admiral bears from the admiral N.N.E. 6 miles, and a look-out steamer is S.E.b.S. 10 miles. The admiral, suspecting the enemy to be in the S.E. quarter, directs the vice-admiral, by signal, to take station 3 miles S. E. of him, and the look-out steamer due east 5 miles; at the same time he sends several steamers from the main body to reinforce the vice-admiral at the appointed station. Now, supposing the admiral continue to steer due east, 6 miles an hour, and the vice-admiral to increase his speed to 10 miles, and the look-out steamer to 14 miles an hour, it is required to find the course and distance of the look-out steamer, and also the course and distance of the vice-admiral, so that they may reach their several stations in the shortest time. Required also the time in reaching their stations, the rate at which the reinforcement must steam

to reach the station at the same time as the vice-admiral, and also its course and distance.*

*Ans.* Vice-admiral's course is S.E.b.S. nearly; distance, 9·42 miles. Look-out steamer's course is N.N.E. nearly; distance, 8·94 miles. Reinforcement's course is E.b.S.$\frac{1}{2}$S. nearly; distance, 8 miles. The vice-admiral and reinforcement reach their station in 56 minutes, and the look-out steamer in 38 minutes. The reinforcement's rate of steaming, $8\frac{1}{2}$ miles an hour.

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## SECTION II.

### ASTRONOMICAL AND NAUTICAL PROBLEMS.

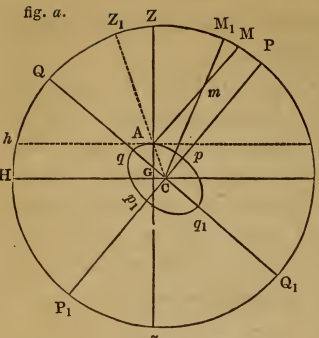
Spherical Trigonometry derived its origin from the computations necessary in astronomy, and its principal applications are still furnished by the same science.

To assist the student in understanding the nature of the following problems, we will give in this place a few astronomical and nautical terms and definitions.

*Definitions in Astronomy.*

The astronomer conceives all the heavenly bodies to be contained within a hollow sphere of great but indefinite magnitude; he supposes this sphere (the interior surface of which is called the *celestial concave*) to have the same center as the earth; and in all applications of spherical trigonometry, he employs, instead of the real position of any of the heavenly bodies, the point in which the celestial concave is cut by a straight line drawn through the *centers* of the earth and of the body;

* If a geometrical construction of this problem be made on a large scale, the course, &c. so found will be sufficiently correct for practical purposes.




this point is called the *true* place of the heavenly body. Thus, fig. *a*, let $QZPQ_1P_1$ represent the celestial concave, *m* a heavenly body, C the center of the earth; join C*m*, and produce it to cut the celestial concave in $M_1$; then $M_1$ is the true place of the heavenly body *m*. Let A be the place of a spectator on the surface of the earth: join A*m*, and produce it to the celestial concave at M; then M is called the *apparent* place of *m*.

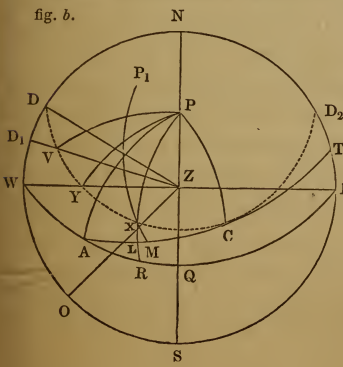
The extremities of the axis on which the celestial concave appears to revolve, in consequence of the earth's diurnal motion, are called the *poles of the heavens;* thus, $PP_1$ represents the axis of the heavens, P and $P_1$ are the poles: the points *p* and $p_1$, in which the earth's surface, $pqp_1q_1$, is cut by this axis, are called the *poles of the earth*. That great circle on the surface of the earth which is equidistant from each of its poles, is called the *terrestrial equator;* in the figure, $q\,q_1$ represents the plane of the terrestrial equator. The terrestrial equator extended to the celestial concave, as $QQ_1$, forms the *celestial equator*. A circle touching the earth where the spectator stands, and extending to the celestial concave, is called the *visible* horizon, and a circle parallel to the visible horizon which passes through the center of the earth and extends to the celestial concave is called the *rational* horizon; thus, *hr* represents the visible, and HR the

rational horizon of the spectator at A. These two circles, however, form one and the same in the celestial concave: thus, the points R and *r* in the figure must be supposed to coincide. This may be readily conceived, when we consider that the distance of any two points on the surface of the earth will make no sensible angle at the celestial concave; therefore, either of these two circles is to be understood by the word *horizon*. Of the poles of the horizon of any place, that which is over the place is called the *zenith*, and the other the *nadir*, as Z and $z_1$ in fig. *a*.

Great circles passing through the zenith are called *circles of altitude*, or *vertical circles;* the circle of altitude passing through the poles of the heavens is also called the *celestial meridian;* the points of the horizon through which the celestial meridian passes are called the north and south points; and a circle of altitude at right angles to the meridian is called the *prime vertical;* this last circle cuts the horizon in the east and west points.

These circles will be more clearly understood by means of fig. *b*, in which N.W.S.E. represents the horizon of a spectator whose zenith is supposed to be Z; N. S. E.W. the north, south, east, and west points of the horizon. NZS is a circle of altitude passing through the pole, and therefore repre-

fig. *b*.



sents the celestial meridian: ZD, ZO, &c. are circles of altitude, or vertical circles, and WZE is the prime vertical.

Since the horizon and celestial equator are both perpendicular to the celestial meridian, the points where the horizon and celestial equator intersect each other must be 90° distant from every part of the meridian [Part II. Art. (65), Cor. 3]; that is, the celestial equator cuts the horizon in the east and west points; draw the curve EQW to cut the horizon in the east and west points; this will represent the celestial equator. Since the poles of the heavens are 90° distant from the equator, take QP=90°. P is called the elevated pole, or the one above the horizon, and NP is the altitude of the pole.

The *distance*, ZQ (figures *a* and *b*), of the zenith from the equator represents the *latitude of the spectator;* this may be more clearly seen in fig. *a*, where PZQH represents the plane of the celestial meridian, Z the zenith of the spectator at A, *hr* or HR his horizon, P the elevated pole, $QQ_1$, drawn at right angles to $P_1P$, represents the plane of the celestial equator, and the angle ZGQ (the earth being considered as a spheroid) is the latitude of the spectator; but since the distance, ZQ, of the zenith from the equator measures the same angle, this arc is taken to represent the latitude.* ZP (figures *a* and *b*), the complement of $ZQ_1$, is called the *colatitude*.

* If CA be joined, and produced to cut the celestial concave in $Z_1$, the arc $Z_1Q$ is called the latitude *on the sphere*, or the reduced latitude. The formula given in p. 38, *Trigonometry*, Part I. enables us to find the difference between the true and reduced latitude.

The altitude of the pole is easily shewn to be equal to the latitude of the spectator:

for alt. NP = 90° − PZ = 90° − colat.
= 90° − (90° − lat) = latitude.

The sun, in consequence of the earth's motion in its orbit, appears to move eastward in the celestial concave, and, in the course of a year, to describe, among the fixed stars, a great circle; this circle is called the *ecliptic*. It is inclined to the celestial equator at an angle of about 23° 28′, called the *obliquity* of the ecliptic. In fig. *b*, ACT represents a part of the ecliptic, and the angle MAR the obliquity. The points in which the equator and ecliptic intersect are called the *first point of Aries* and *first point of Libra;* the former, being the point where the sun crosses the equator northward, is called the *vernal equinoctial point*, and the latter the *autumnal* equinox. Great circles passing through the poles of the heavens are called circles of *declination*, and great circles passing through the poles of the ecliptic are circles of *latitude;* thus, in fig. *b*, PR, PA, &c. are circles of declination, and if $P_1$ represent the pole of the ecliptic, $P_1M$ is a circle of latitude. *Parallels of declination* and *latitude* are small circles parallel respectively to the celestial equator and ecliptic.

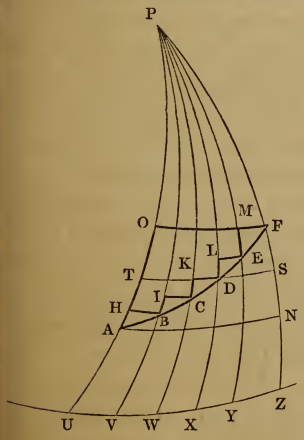
The *declination* and *right ascension* of a heavenly body may be defined thus: the *declination* of a heavenly body is the arc of the circle of declination passing through its place in the celestial concave, intercepted between this place and the celestial equator; the *right ascension* of a heavenly body is the arc of the celestial equator intercepted between the first point of Aries and the circle of declination passing through the place of the body in the

celestial concave. Or, it is the angle at the pole of the heavens between the circles of declination passing through the first point of Aries and the place of the heavenly body; the arc or angle being measured round the equator or the pole from west through south to east, or in the direction of the earth's motion round the sun; thus, in fig. *b*, if X be a heavenly body, and PXR a circle of declination passing through its place, XR is its declination, and AR, or the angle APR, its right ascension. In like manner, if a circle of latitude be drawn through any point in the celestial concave, the part of it between the point and the ecliptic is called the *latitude* of the point, and the arc of the ecliptic extending eastward from the first point of Aries to the circle of latitude is called the *longitude* of the point; thus, the latitude of X is XM, and longitude AM. The *altitude* of a heavenly body is the arc of a circle of altitude intercepted between the place of the body and the horizon; thus, XO is the altitude of X. The *azimuth* of a heavenly body is the arc of the horizon intercepted between the north or south points and the circle of altitude passing through the place of the body; or, it is the corresponding angle at the zenith between the celestial meridian and the circle of altitude passing through the body; thus, SO or NO, or the angles OZS or OZN, is the azimuth of X. The *amplitude* of a heavenly body is the distance from the east at which it rises, or the distance from the west at which it sets, these arcs or distances being measured on the horizon; thus, the amplitude of X is the arc WD or $ED_2$ (the dotted line $D_2XD$ being the arc of a *parallel* of declination described by X from rising at $D_2$ to setting at D). The *hour-angle* of a heavenly body is the

angle at the pole between the celestial meridian and the circle of declination passing through the place of the body; thus, ZPX is the hour-angle of X. If L be the place of the sun *west* of the meridian, its hour-angle ZPL is called *apparent* time; but when the sun is *east* of the meridian, as at C, then apparent time is found by subtracting the hour-angle ZPC from 24 hours.

### *Definitions in Navigation.*

Let A and F represent two places on the surface of the earth (considered as a sphere), PU, PZ, their meridians, P the pole, and UZ an arc of the equator. Through A and F draw a curve line, AF, cutting all the intermediate meridians, PV, PW, &c. *at the same angle.* This common angle is called the *course* from A to F, and the arc AF (in nautical miles) is called the *distance.* Draw the parallels of latitude AN and FO; the arc AU is the latitude of A, and FZ the latitude of F; UZ, or the angle APF, is the difference of longitude between A and F. The arc AO is the difference, or as it is called in Navigation, the *true difference of latitude* between A and F.



Suppose the intermediate meridians, PV, PW, &c. to be drawn through points B, C, &c. taken on the arc AF *indefinitely near to one another*, and through B, C, &c. suppose arcs of parallels BH, CI, &c. to be drawn; on this supposition the elementary triangles ABH, BCI, &c. may be considered as *right-angled plane triangles*, of which the sum of the sides AB, BC, &c. is the distance, the sum of the sides AH, BI, &c. is equal to the true difference of latitude, and the sum of the sides BH, CI, &c. is called in Navigation the *departure*.

The chart used at sea for marking down the ship's track and for other purposes, exhibits the surface of the globe on a plane on which the meridians are drawn parallel to each other, and therefore the parts BH, CI, DK, &c. arcs of parallels of latitude, are increased and become equal to the corresponding parts of the equator UV, VW, &c. Now, in order that every point on this plane may occupy the same relative position with respect to each other that the points corresponding to them do on the surface of the globe, the distance between any points A and O, and A and F must be increased in the same proportion as the distance FO has been increased. The true difference of latitude, AO, is thus projected on the chart into what is called the *meridional difference* of latitude, and the departure, BH+CI+DK+ . . . into the difference of longitude. A chart constructed in this manner is called a Mercator's Chart. From these definitions and principles are deduced the fundamental formulæ in Navigation given in p. 38, *Trig.* Part I.; and these expressed in words form the common rules of Mercator and parallel sailing. The investigation of these formulæ and rules will be given in the present volume.

## PROBLEMS IN ASTRONOMY, ETC.

[102]. In latitude 50° 48′ N. the altitude of the sun was 46° 20′ (west of meridian), and, by the Nautical Almanac, its declination at the time of observation was 23° 27′ 45″ N.; required the azimuth.

*Ans.* N. 111° 51′ W.

[103]. In latitude 50° 48′ N. the altitude of the sun was 46° 20′ (west of meridian), and its declination 23° 27′ 45″ N.; required the apparent time of observation.

*Ans.* 2h. 57m. 16 sec.

[104]. The azimuth of a heavenly body was observed to be N. 111° 51′ W., its altitude at the same time was 46° 20′, and declination 23° 27′ 45″ N.; required the apparent time. *Ans.* 2h. 57m. 16 sec.

[105]. What is the altitude of a star whose hour-angle is 38° and declination 16° N. at a place in latitude 50° 48′ N.? *Ans.* 43° 49′.

[106]. In latitude 50° 48′ N., when the sun's declination was 12° 29′ N. and hour-angle 2h. 53m. 1sec. A.M.; required the azimuth. *Ans.* N. 121° 47′ E.

[107]. Given the sun's meridian altitude = 70° (zenith north of the sun) and its declination 20° N.; required the latitude of the place. *Ans.* 40° N.

[108]. Given the sun's meridian altitude = 70° (zenith north) and its declination 5° S.; required the latitude. *Ans.* 15° N.

[109]. Given a star's meridian altitude = 70° (zenith south) and declination 25° N.; required the latitude.

*Ans.* 5° N.

[110]. Given the sun's meridian altitude 30° (zenith south) and declination 10° N.; required the latitude.

*Ans.* 50° S.

[111]. Given the latitude of the spectator 40° N. and the declination of a heavenly body 20° N.; required its meridian altitude (zenith north of the body).

*Ans.* 70°.

[112]. Given the sun's meridian altitude 30° (zenith south of the sun) at a place in latitude 50° S.; find its declination. *Ans.* 10° N.

[113]. The meridian altitude of a star at a place on the equator is 57°; find its declination (zenith north of star). *Ans.* 33° S.

[114]. The latitude of a spectator is equal to half the sum of the meridian altitudes of a circumpolar star above and below the pole; required a proof. Given the altitudes 20° and 70° at its inferior and superior transits respectively; required the latitude.* *Ans.* 45° N.

[115]. A circumpolar star passes the zenith of a place, and its altitude at the inferior transit is 20°; required the latitude. *Ans.* 55° N.

[116]. The altitude of a circumpolar star at its inferior transit is equal to its zenith distance at its superior transit; required the latitude.

*Ans.* 45° N.

[117]. In latitude 60° N. what are the altitudes of a star at the inferior and superior transits, its declination being 40° N. *Ans.* 70° and 10°.

[118]. What is the declination of a star that passes the zenith of a place in latitude 50° 48′ N., and what will be its altitude at the inferior transit?

*Ans.* Decl. 50° 48′ N.; alt. 11° 36′.

[119]. What is the latitude of a place in which the

* The latitude and declination in the following problems are *north*, unless the contrary is expressed.

sun just touches the horizon without setting, when its declination is 23° 28′ N.? *Ans.* 66° 32′ N.

The following expressions will be found useful for certain positions of the heavenly body. They are deduced from Napier's Rules for right-angled and quadrantal triangles (see pp. 66, 70, *Trig.*, Part I.).

1. When the heavenly body is on the *prime vertical*, prove the following formulæ:

$\alpha$. cos. hour-angle = cot. lat. × tan. declination.
$\beta$. sin. decl. = sin. lat. × sin. altitude.
$\gamma$. cos. alt. = sin. hour-angle × cos. decl.
$\delta$. cos. lat. = cot. hour-angle × cot. alt.

2. When the heavenly body is *six hours*, or 90°, from the meridian:

$\varepsilon$. sin. alt. = sin. decl. × sin. lat.
$\zeta$. cos. azimuth = cot. lat. × tan. alt.
$\eta$. cos. lat. = cot. decl. × cot. azimuth.
$\theta$. cos. decl. = sin. azimuth × cos. alt.

3. When the heavenly body is in the *horizon:*

$\iota$. cos. hour-angle = − tan. lat. × tan. decl.
$\kappa$. sin. decl. = cos. lat. × sin. amplitude.
$\lambda$. sin. lat. = − cot. amplitude × cot. hour-ang.
$\mu$. cos. amplitude = cos. decl. × sin. hour-angle.

[120]. Given the sun's altitude 30° when due west, and its declination 20° N.; required the latitude.

*Ans.* 43° 9′ 15″ N.

[121]. Given the sun's declination 23° 27′ 45″, and latitude of the place 50° 48′ N., to find his altitude, and the time when he is on the prime vertical.

*Ans.* Alt. 30° 55′; time, 4h. 37m. 4 sec.

[122]. Two stars are due east at the same time at a place whose latitude is 50°48′ N., and their altitudes are 20° and 40°; find the difference of their hour-angles.

*Ans.* 59m. 56 sec.

[123]. Given the sun's altitude 18°45′ at six o'clock, and declination 20° 4′ N., to find the latitude.

*Ans.* 69° 31′ 40″ N.

[124]. Given the latitude 50° 48′ N., and sun's declination 23° 27′ 45″ N., to find his altitude and azimuth at six o'clock.

*Ans.* Alt. 17° 58′ 15″; azimuth, N. 74° 39′ 30″ E.

[125]. What will be the apparent time of rising of the sun at a place in latitude 50° 48′ N. when its azimuth is S. 80° E. (neglecting the effects of refraction, &c.)?

*Ans.* 6h. 31m. 7 sec.

[126]. Given the sun's amplitude W. 37° 30′ N. and decl. 15° 12′ N.; required the latitude.

*Ans.* 64° 29′ 15″ N.

[127]. Given the latitude of the place 50°48′ N., and the sun's decl. 18°28′ N.; required his amplitude, time of rising or setting, and the length of the day and night.

*Ans.* Amp. E. 30°4′ 30″ N.; sun rises at 4h. 23m. 19 sec. A.M.; length of day, 15h. 13m. 22 sec.

[128]. Where will the sun rise in latitude 50° 48′ N. when the day is 14 hours long? *Ans.* E. 19° 5′ N.

[129]. Given the sun's altitude 22° 56′, and hour-angle 3h. 0m., when the declination is nothing, to find the latitude. *Ans.* 56° 33′ 30″ N.

[130]. Given the sun's altitude, declination, and azimuth; to find the latitude.

EXAMPLE.

Alt.=42°30′; decl.=22° 10′ N.; azimuth=S. 57°45′ W.

*Ans.* Lat. 59° 4′ N.

[131]. Given the sun's altitude 37° 20′, hour-angle 2h. 15m., and decl. 22° 30′N. to find the latitude (zenith north of the sun). *Ans.* Lat. 71° 31′ N.

[132]. Given the right ascension of the sun (RA), and obliquity of the ecliptic ($\omega$), to find his longitude and declination.

EXAMPLES.

1. RA=4h. 10m. 20 sec.; $\omega$=23° 27′ 45″.
   *Ans.* Long. 64° 33′ 15″; decl. 21° 4′ 15″ N.
2. RA=17h. 10m.; $\omega$=23° 27′ 45″.
   *Ans.* Long. 258° 30′ 15″; decl. 22° 58′ S.

[133]. Given the right ascension and declination of a heavenly body and the obliquity of the ecliptic, to find its latitude and longitude.

EXAMPLES.

1. RA = 2h. 59m. 37 sec.; decl. = 21° 27′ 48″ N.; $\omega$=23° 27′ 45″.
   *Ans.* Lat. 4° 15′ N.; long. 48° 37′ 30″.
2. RA=16h. 14m.; decl.=25° 51′ N.; $\omega$=23° 27′ 45″.
   *Ans.* Lat. 46° 6′ 15″ N.; long. 234° 36′ 30″.

[134]. Two places have the same latitude, namely, 45° N., and their difference of longitude is 10° 36′; required their distance.

*Ans.* 449·25 nautical miles.

[135]. Required the distance from Portsmouth to Buenos Ayres.

Lat. Portsmouth 50° 48′ N.; long. 1° 6′ W.
Lat. Buenos Ayres 34 37 S.; long. 58 24 W.

*Ans.* 5949·8 nautical miles, or 6847·2 English miles ($69\frac{1}{20}$ to a degree).

*Sailing on or near to a Great Circle.*

The shortest distance between two known places is the arc of a great circle passing through them. This distance may be found as in the last example. The practical inconvenience of sailing on a great circle arises from the necessity of continually altering the course. It is for this reason that Rules are given for sailing from one place to another so that the course may be constant, although the distance sailed is not the shortest between the two places. When the distance between the two places is considerable, as between Liverpool and New York, the following method of approximating to the shortest distance may be adopted with advantage.

1. Compute the shortest distance as in the last example.

2. Take two or more points on this arc, and find the latitude and longitude of those points.*

3. Find the course and distance from the place sailed from, to the nearest point marked on the arc, and thence to the next point, and so on.

The distance described by the ship on these curves, will not differ much from the shortest distance; and thus the advantage of sailing on one course is combined with that of shortening the distance.

EXAMPLES IN GREAT-CIRCLE SAILING.

1. Compute the shortest distance between Liverpool and New York.

* The points should not be farther apart than 1000 miles. The nearer they are taken to each other the less will be the difference between the sum of the distances run and the shortest distance.

Lat. Liverpool, 53° 25′ N ; long. 2° 59′ W.
Lat. New York, 40 42 N.; long. 73 59 W.
*Ans.* 2872 miles.

2. Take a point, A, on this arc, distant from Liverpool 872 miles; take another point, B, distant from A = 1000 miles; then from B to New York = 1000 miles. Find the latitude and longitude of the points A and B.

*Ans.* Lat. A, 54° 39′ N.; long. 27° 46′ W.
„ B, 50 0 N.; „ 54 9 W.

3. Required the course and distance (by Mercator's sailing [p. 39, *Trig.*, Part I.]) from Liverpool to A, from A to B, and from B to New York.

*Ans.* Course and distance from:
Liverpool to A, N. 85° 9′ W.; dist. 876 miles.
A to B . . . . S. 73 54 W.; „ 1006 „
B to New York, S. 56 11 W.; „ 1002 „

4. Required the difference between the distance, by great-circle sailing, from Liverpool to New York, and the distance by Mercator's sailing. See Ex. 2, p. 39, *Trig.*, Part I. *Ans.* 107 miles.

[136]. Required the distance of the moon from $\alpha$ Leonis (Regulus); the right ascension and declination of the former being 0° 32′ 45″ and 5° 19′ S., and of the latter 148° 18′ 45″ and 13° 10′ 15″ N. *Ans.* 147° 16′.

[137.] The meridian altitude of $\alpha$ Leonis (declination 13° 10′ N.) under the north-pole was observed to be 6° 10′; required the latitude. *Ans.* 83° N.

[138]. Given the sun's longitude 202° 24′ 15″, and the moon's latitude and longitude 4° 54′ 30″ N. and 89° 25′ 30″; required their distance. *Ans.* 112° 53′ 30″.

[139]. If a ship from latitude 50° 10′ N. start on a

S.W. course, and sail 100 miles on a great circle, what will be her last course? *Ans.* S. 43° 38′ W.

[140]. The latitude of a place A is 40° N., of B 50° N., and their distance from each other 20°, the longitude of A is 15° E.; required the latitude and longitude of another place C to the north of and 20° distant from A and B.

*Ans.* Lat. 59° 37′ N.; long. 21° 13′ E. or 8° 47′ E.

[141]. The distances of a comet from Aldebaran and Regulus were observed to be 40° 12′ and 51° 36′ respectively; required its latitude and longitude, the respective latitudes of the two stars being 5° 28′ 45″ S. and 0° 27′ 30″ N., their longitudes 67° 12′ 15″ and 147° 15′ 30″, and the comet being south-east of the arc of a great circle joining them.

*Ans.* Lat. 28° 0′ 15″ S.; long. 102° 19′.

[142]. Required the beginning of morning and the end of evening twilight, at a place in latitude 54° 36′ N., the sun's declination being 8° 30′ N. (twilight being supposed to begin and end when the sun is 18° below the horizon). *Ans.* 2h. 46m. A.M., 9h. 14m. P.M.

[143]. Suppose two altitudes of the sun observed in the forenoon in the same place, at the interval of an hour and a half, to be 28° 40′ and 39° 50′; required the latitude of the place, the declination being 23° 26′ N. at both observations. *Ans.* Lat. 59° 17′ 15″ N.

[144]. The altitudes of $\alpha$ Hydræ and Regulus (east of meridian) were at the same time 40° 44′ and 45° respectively, the right ascension and declination of the former being 9h. 16m. and 7° 37′ S., of the latter 9h. 53m. and 13° 9′ N.; find the latitude.

*Ans.* 26° 23′ N.

[145*a*]. The difference of longitude between two places in the same latitude, namely, 33° 51′ S., is 136° 10′; how much shorter is the distance between them on the arc of a great circle, than on their common parallel; and what is the highest latitude attained by a ship sailing from one to the other on the arc of a great circle?

*Ans.* Difference of distances, 737·6 English miles; highest lat. 60° 54′ S.

[146*a*]. What is the highest latitude attained by a ship sailing on the arc of a great circle from Port Jackson to Cape Horn, their latitudes being 33° 51′ and 55° 58′, and the difference of longitude 140° 27′?

*Ans.* 72° 41′.

[147]. Required the sun's azimuth and depression below the horizon at 7h. P.M., when the declination was 10° 15′ S., and the latitude of the place 50° 48′ N.

*Ans.* Azimuth, N. 84° 53′ W.; depression, 17° 24′.

[148]. Determine the bearing or azimuth of the two stars Aldebaran and Pollux when on the same vertical circle, the latitude of place being 25° N.; the R.A. and declination of former star being 4h. 26m. 46s. and 16° 11′ N.; of the latter 7h. 35m. 13s. and 28° 24′ 26″ N.

*Ans.* S. 75° 5′ W.

[149.] In a certain latitude (zenith N.) the moon's true altitude was 18° 2′ 30″ (east of meridian) in the same vertical circle with a star whose right ascension and declination were 9h. 59m. 42s. and 12° 45′ 45″ N., the moon's R.A. and declination being 12h. 35m. 54s. and 1° 42′ 30″ S.; required the latitude.

*Ans.* 19° 55′ 30″ N.

[150*a*]. The altitude of a star when due east was 20°, and it rose E.b.N.; required the latitude.

*Ans.* 29° 42′ N.

[151*a*]. The altitude of a star when due east was 10°, and when due south was 40°; find the latitude of the place. *Ans.* 58° 31′ N.

[152*a*]. Given the altitude of the sun when due west and at six o'clock, to find the latitude and declination.

EXAMPLE.

Altitude when west = 27° 24′; at 6h. = 14° 43′ 30″.

*Ans.* Latitude, 48° N.; declination, 20° N.

[153*a*]. Given the sun's altitude at six o'clock, and its amplitude, to find the latitude and declination.

EXAMPLE.

Altitude at 6h. = 14° 43′ 30″; amplitude = 30° 44′ 30″.

*Ans.* Latitude, 42° or 48°; declination, 22° 20′ or 20°.

[154*a*]. Given the sun's altitude at six o'clock, and the hour-angle when setting, to find the latitude and declination.

EXAMPLE.

Altitude at 6h. = 14° 43′ 30″; $h$ = 7h. 35m. 22s.

*Ans.* Latitude, 48° 1′; declination, 20°.

[155*a*]. Given the times at which the sun sets and is west on the same day, at a particular place, to find the latitude of the place, and the declination.

EXAMPLE.

Hour-angle when W. = 4h. 43m. 28s.

setting = 7h. 35m. 22s.

*Ans.* Latitude, 48° N.; declination, 20° N.

[156*a*]. Given the sun's declination and the interval between the times at which he is west and sets, to find the latitude.

EXAMPLE.

Declination = 20° N.; interval = 2h. 51m. 54s.

*Ans.* Latitude, 48° N. or 42° N.

[157*a*]. Given the amplitude of the sun, and the azimuth at six o'clock, to find the latitude and declination.

EXAMPLE.

Ampl.=W. 30° 44′ 30″ N.; azimuth=N. 76° 18′ 45″ W.

*Ans.* Latitude, 48° N.; declination, 20° N. nearly.

[158*a*]. Given the sun's meridian altitude, and his altitude at six o'clock; to find the latitude and declination.

EXAMPLE.

Mer. alt. =62°; alt. at 6h.=14° 43′ 30″.

*Ans.* Latitude, 48° 0′ 20″ N.; declination, 20° N.

[159*a*]. Given the sun's meridian altitude, and the hour-angle when rising; to find the latitude and declination.

EXAMPLE.

Mer. alt.=56°; hour-angle=7h.

*Ans.* Latitude, 47° 23′ 30″.

[160*a*]. Given the interval between the times at which the sun is west and sets at a place whose latitude is known; to find the declination.

EXAMPLE.

Lat.=48° N.; interval = 2h. 51m. 54s.

*Ans.* Declination, 20° N.

[161*a*]. At a given place, to find the greatest azimuth of a heavenly body, whose declination is greater than the latitude of the place; to find also the time and altitude on a given day, when the heavenly body will have the greatest azimuth, and when consequently it will appear to move perpendicularly to the horizon.

EXAMPLE.

In lat. 20° N. when the sun's declination is 23° 28′ N.; required the time and altitude when its azimuth is the greatest, and also its greatest azimuth.

*Ans.* Time, 9h. 47m. 53s. A.M. (app. time); altitude, 59° 11′ 30″; greatest azimuth, N. 77° 28′ E.

[162*a*]. In latitude 20° N. when the sun's declination is 23° 28′ N.; required the time when the sun will appear stationary in azimuth, the period during which the shadow moved in a contrary direction, and the number of degrees it appeared to go back.

*Ans.* Time, 9h. 47m. 53s.; period, 4h. 24m. 15s.; shadow went back 12° 32′ 30″.

[163*a*]. When the sun's declination was 10° 15′ N. and that of the moon was 12° 46′ S., both were observed to rise at the same time; required the latitude of the place and the time of the observation, the difference of their R.A.'s being 1h. 53m. 42s.

*Ans.* App. time, 5h. 9m. 40s. A.M.; lat. 50° 18′ 20″ N.

[164*a*]. On what days of the year is the sun on the horizons of Dublin and Pernambuco at the same instant, their respective latitudes being 53° 21′ N. and 8° 13′ S., and their longitudes 6° 19′ W. and 35° 5′ W?

*Ans.* The four days in the year when the sun's declination is 18° 6′.

[165]. ABCD is a square field, each of whose sides is 100 yards. In the middle of the field stands an obelisk 60 feet high; find the altitude of the sun when the shadow of the obelisk just reaches the corner of the square. *Ans.* 15° 47′.

[166*a*]. At noon on the shortest day, the shadow of a perpendicular stick was 7 times as long as its shadow at noon on the longest day; required the latitude, declination 23° 28′.

*Ans.* Sin. $2l = \frac{4}{5}$ sin. $2d$; $l = 38°\ 27'\ 45''$.

[167*a*]. Compare the lengths of the shadow of a per-

pendicular stick at noon in latitude 45° N. on the two days when the sun's declination was 15° N. and 15° S.

*Ans.* Lengths of shadows as 3 : 1.

[168*a*]. In latitude 33° 30′ N. and decl. 10° 15′ N., I observed that my shadow bore to my height the proportion of 5 : 3 ; required the altitude and hour-angle of the sun. *Ans.* Alt. 30° 58′; $h$=3h. 58m. 4s.

[169*a*]. The length of the shadow of a perpendicular object was 4 feet, and its longest when sloping was 5 feet; required the sun's altitude. *Ans.* 36° 52′ 15″.

[170*a*]. The elevation of a cloud was observed to be 20°, at the same time the sun's altitude was 22°, the sun and cloud being in the same plane with the observer, and his distance from the shadow 400 yards; required the height of the cloud. *Ans.* 1468 yards.

[171*a*]. In latitude 45° N., the meridian altitude of the sun = 30°; shew that the tangent of quarter the length of the day $= \frac{1}{\sqrt[4]{3}}$.

[172*a*]. At a certain place the sun rose at 7 h. A.M., and its meridian zenith distance was twice the latitude ; required the latitude. *Ans.* Lat. 26° 58′.

[173*a*]. In latitude 45° N., the sun rose at 4h. A.M. ; shew that the tangent of the meridian altitude = 3.

[174*a*]. In latitude 50° N. when the sun's declination is 5° 38′ N. ; required the time it will take the body of the sun to rise out of the horizon, its semidiameter being 16′. *Ans.* 3m. 19s.

[175*a*]. Required the time the sun's semidiameter will take to pass the meridian, the declination being 23° 4′ and semidiameter 16′ 17″·3. *Ans.* 1m. 11s.

[176*a*]. In latitude 59° 6′ N. and long. 6° 15′ E. ob-

served a point of land bearing N.E., and after sailing E.N.E. 6 miles the point bore N.$\frac{1}{2}$W.; required the latitude and longitude of the point.

*Ans.* Lat. 59° 11′ 15″; long. 6° 25′ 13″ E.

[177*a*]. In latitude 45° N., required the difference in the lengths of the longest and shortest days (declination 23° 28′). *Ans.* 6h. 51m. 40s.

[178*a*]. In what latitude N. will the difference between the longest and shortest days be just 6 hours?

*Ans.* Lat. 46° 24′ N.

[179*a*]. At a certain place, when the sun's declination was 10° N., it rose an hour later than when it was 20° N.; required the latitude. *Ans.* Lat. 52° 27′ N.

[180*a*]. In what latitude north will the shortest day be just one-third the longest (declination 23° 29′)?

*Ans.* 58° 27′ N.

[181*a*]. And in what latitude will the shortest day be just $\frac{3}{5}$ths the longest? *Ans.* 41° 24′ N.

[182*a*]. If a ship sail from a certain place $a$ miles due east, then $a$ miles due south, and then $b$ miles due west, and reach the same longitude; required the latitude of the place arrived at.

EXAMPLE.

$a=100'$, and $b=150'$. *Ans.* 85° 0′ 30″.

[183*a*]. Given the apparent and true altitude of a heavenly body, its declination, and observed distance from a terrestrial object, to find the true bearing of the object from a given station.

EXAMPLES.

1. When the object is in the horizon.

In latitude 7° 51′ N. the observed altitude of the sun's lower limb was 10° 30′, and observed distance of

his nearest limb from a well-defined point of land on the same level with the eye and to the right of the sun was 95° 16′; index correction of the altitude sextant was 50″—, and that of the other was 1′ 10″+; the correction for height of the eye (14 feet) in taking the sun's altitude was 3′ 41″—; required the true bearing of the point of land; the sun's declination being 22° 24′ S. and semidiameter 15′ 45″. *Ans.* Bearing, N. 19° 0′ 30″ W.

2. When the object is elevated above the level of the eye it is necessary to observe its altitude.

In the preceding example, suppose the object observed to be a mountain the altitude of whose summit is 10°, required the true bearing of that point.

*Ans.* N. 17° W.

## TRIGONOMETRICAL SURVEYING. MEASUREMENT OF AN ARC OF THE MERIDIAN.

The system of rules and operations by which the relative position of any number of points in a tract of country may be determined, so that it may be delineated on a plane surface, is called *Trigonometrical Surveying*. When the extent of country is not great, the subject involves little difficulty; but when a kingdom such as Britain or France is to be surveyed, the aid of astronomy and other branches of natural philosophy is required. In a survey, the most remarkable objects, such as the summit of hills, spires, towers, &c. must be chosen as *stations*, and, if necessary, marked by signals. These must be considered as joined by straight lines, forming a chain of triangles, connecting each with all the others. The sides of the triangles should be as long

as possible, so as to admit of the stations at any two of the angles being seen from the third; the nearer each triangle approaches to the equilateral form the better. Supposing a proper disposition of the triangles to have been made, when their angles are known, if a side of any one of them were also known, then the sides of all the others might be found by calculation, and a plan of the country constructed.

A small extent of the earth's surface may be regarded as a plane, and lines perpendicular to it as parallel to one another. In an extensive survey, however, such as that of England, the curvature of the earth must be taken into the account, and then its *figure* and *magnitude* enter as elements into all the calculations. This connexion between the figure of the earth and the magnitude and position of lines traced on its surface affords conversely the means of determining the former when the latter are known; so that such surveys, besides their immediate object, are applicable to the solution of the still more important problem of finding the magnitude and figure of the earth itself.

When a tract to be surveyed has been covered with a series of triangles, so as to connect the principal points, and all the angles of each and a side of one are known, the sides of all the triangles may be found by calculation, and a plan made by constructing the triangles on the sides of each other; but in a plan so constructed any error made in the value of one of the sides or angles will produce corresponding errors in all the others.

To avoid as much as possible this source of error, we may determine the position of a side of one of the triangles with respect to the meridian, by compass, or more accurately by astronomical observations (see Problem

183); and then calculate the distances of all the stations from such meridian, and also the meridional distance of one station from the other; from these distances the position of each station may be laid down in the plan independently of the others, and also the direct distance between any two points may be easily found. The manner of proceeding will appear from the following Problem.

[184]. (Fig. 59.) Let A, B, C, D, E, F, be six stations connected by four triangles ABC, BCD, BDE, EDF; the angles are:

| | |
|---|---|
| BAC = 79° 20′ | CBD = 39° 20′ |
| ABC = 51 31 | BCD = 69 28 |
| ACB = 49 9 | BDC = 71 12 |
| DBE = 45 28 | EDF = 62 3 |
| BDE = 72 3 | DEF = 52 25 |
| BED = 62 29 | DFE = 65 32 |

A side, AB, of one of the triangles is 4213 yards, and it makes with the meridian, NS, an angle SAB = 62° 52′ at the point A; and the station F makes at the point S an angle ASF with the meridian = 52° 40′. It is required to find the points in which perpendiculars from the stations will cut the meridian, and the length of each perpendicular.

SOLUTION.

1. Draw B*b*, C*c*, D*d*, E*e*, F*f*, perpendicular to the meridian, and B*n*, D*q*, parallel to it, forming the right-angled triangles AB*b*, BC*m*, BD*n*, ED*p*, FD*q*. Because the angles of the four triangles ABC, BCD, BDE, DEF, are given, and also AB, a side of one of them, the five lines AB, BC, BD, DE, DF, may be found from AB and each other (their logarithms are:

AB = 3·624591, BD = 3·733559, DF = 3·578533,
BC = 3·738255, and DE = 3·638690).

2. In the right-angled triangle AB$b$, the side AB (or its log.) and the angle BA$b$ = 62° 52′ are known, hence we find A$b$ = 1921·4, B$b$ = 3749·3 yards.

3. If from AB$m$ = 117° 8′ (the supplement of BA$b$) the angle ABC = 51° 31′ be taken, there remains CB$m$ = 65° 37′; therefore in right-angled triangle CB$m$ the angles and the log. of BC are now known, hence we find B$m$ = 2259·6 and C$m$ = 4985·2.

4. And if from CB$m$ = 65° 37′ CBD = 39° 20′ be subtracted, there remains DB$n$ = 26° 17′; therefore, in the right-angled triangle DB$n$ the angles and the log. of BD are known, and hence B$n$ = 4854·7 and D$n$ = 2397·6.

5. From BD$q$ = 153° 43′ (supplement of DB$n$) subtract BDE = 72° 3′, and there remains ED$p$ = 81° 40′; then in the right-angled triangle ED$p$ the angles and the log. of DE are known, and thence D$p$ = 630·7, E$p$ = 4306·1.

6. Subtracting EDF = 62° 3′ from ED$p$ = 81° 40′, there remains FD$q$ = 19° 37′; and hence in the right-angled triangle DF$q$ we find D$q$ = 3569·1, F$q$ = 1272·1.

To determine the stations, we have now:

| | |
|---|---|
| A$b$ . . . . . = 1921·4 | B$b$ . . . . . = 3749·3 |
| A$c$ = A$b$ + B$m$ = 4181·0 | C$c$ = C$m$ − B$b$ = 1235·9 |
| A$d$ = A$b$ + B$n$ = 6776·1 | D$d$ = B$b$ − D$n$ = 1351·7 |
| A$e$ = A$d$ + D$p$ = 7406·8 | E$e$ = D$d$ + E$p$ = 5657·8 |
| A$f$ = A$d$ + D$q$ = 10345·2 | F$f$ = D$d$ + F$q$ = 2623·8 |

By these numbers, the position of each station may be laid down with great accuracy in a plan, independently of the others; also the distance between any two may be readily found; for example:

$$\text{CE} = \sqrt{\{(\text{C}c + \text{E}e)^2 + (\text{A}e - \text{A}c)^2\}}$$

*Lastly.* We know by observation that the bearing of F from S is N. 52° 40′ E.; hence in the right-angled triangle F*f*S we find S*f*=200·2 yards; adding this to the line A*f*, we have the length of the meridian line between the stations A and S = 12346·4 yards.

The above method of measuring the meridian line between two distant stations, furnishes, as we have said, the means of finding the approximate length of a degree, and thence, on the supposition of the earth being *an exact sphere*, the magnitude of the earth; for we have only to determine by observation the difference of latitude between the stations A and S, and then divide the whole length of the arc, found as above, by the number of degrees contained in it; the result multiplied by 360 will give the circumference of the earth. Thus, let us suppose the latitude of A has been found by some of the preceding problems to be 49° 48′ 6·4″ N., and that of S 49° 42′ 4·86″ N., and the length of the line AS to be 12346·4 yards; then by a common proportion we easily find that the length of a degree is about 69 miles. Picard, in 1670, by observations and measurements conducted in a manner similar to the above, found the length of a degree of the meridian in latitude 49° N. to be 364881 feet, which differs only 35 yards from what is now considered as the most exact length; an accuracy, however, which, considering the imperfection of his instruments, must be supposed to be quite accidental.

Similar observations and measurements have since been made in different parts of the world, from which it appears that the lengths of a degree of latitude are not invariable, as would be the case if the earth were a sphere; but that they *increase from the equator to the*

*poles;* suggesting to us the figure of an oblate spheriod.

The following table contains the results of such measurements:—

| | Country. | Latitude of middle point of arc measured. | | | Length of 1° in feet. |
|---|---|---|---|---|---|
| | | ° | ′ | ″ | |
| 1 | Sweden . . . . | 66 | 20 | 10 | 365782 |
| 2 | Russia . . . . . | 58 | 17 | 37 | 365368 |
| 3 | England . . . . | 52 | 35 | 45 | 364971 |
| 4 | France . . . . | 46 | 52 | 2 | 364872 |
| 5 | France . . . . | 44 | 51 | 2 | 364535 |
| 6 | Rome . . . . . | 42 | 59 | 0 | 364262 |
| 7 | United States . . | 39 | 12 | 0 | 363786 |
| 8 | Cape of Good Hope . | 33 | 18 | 30 | 364713 |
| 9 | India . . . . . | 16 | 8 | 22 | 363044 |
| 10 | India . . . . . | 12 | 32 | 21 | 363013 |
| 11 | Peru . . . . . | 1 | 31 | 0 | 362808 |

All the observations recorded in the above table (with one exception*) prove that the curvature of the earth must diminish from the equator to the pole: this is sufficient to shew that the earth is not a sphere, and that, in fact, it must approach in form to that of an oblate spheroid.

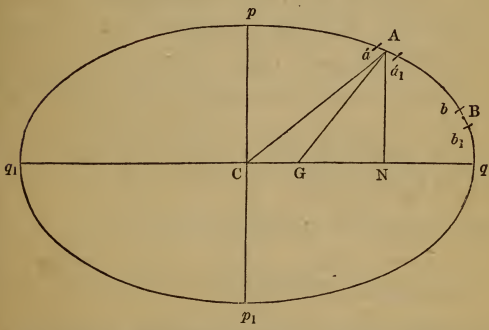
Assuming it to be such a figure, we may compute the lengths of the equatorial and polar diameters from

* The degree measured at the Cape of Good Hope. There is reason, however, to believe that some error has crept into this survey, which will no doubt be discovered, as the observations and measurements of La Caille, who measured this arc, are at this time under revision by the astronomer at the Cape.

the measured lengths of a degree of the meridian in two places differing considerably in latitude.

As this knowledge of the dimensions of the earth is necessary in several important problems in nautical astronomy, we will here investigate an expression for calculating the major and minor diameters $q\,q_1$ and $p\,p_1$, and also the distance CA of any point A from the center of the earth.

*To determine the lengths of the equatorial and polar diameters of the earth.*




Let $p\,q\,p_1\,q_1$ represent a section of the earth passing through the poles, $p\,p_1$; A and B the middle points of two degrees $a\,a_1$, $b\,b_1$, the lengths of which are supposed to be known from the preceding table.

Draw AG a normal at A; then (def. p. 101, *Trig.* Part I.) the angle AG$q$ is the latitude of A.

Let $l$ and $l_1$ represent the latitudes of A and B;

$a$ and $b$ the semi-major and semi-minor axes;

$n$ the normal AG at A;

D and $D_1$ the lengths of $a\,a_1$ and $b\,b_1$, the degrees in latitudes A and B, known from observation.

Then circle of curvature at A$=360\times$D nearly$=2\pi r$

$\therefore$ radius of curvature $r=\dfrac{360\times \mathrm{D}}{2\pi}=57{\cdot}29577\times \mathrm{D}$

Similarly radius of curvature $r_1$ at B$=57{\cdot}29577\times \mathrm{D}_1$

The difference of the equatorial and polar radii is small: call this difference $c$; then an approximate value of $c$ may be found (by neglecting terms involving $\frac{c^2}{a^2}$, &c., which are small), and thence $a$ and $b$. This may be done by expressing the normal $n$ in terms of $a$ and $c$ and the latitude $l$; and then, by differential calculus, $n$ in terms of $r$. A similar expression being found for $r_1$, one of the unknown terms $a\,c$ may be eliminated between the two expressions, and the other determined; thus:

Let the coordinates CN and AN of the point A be denoted by $x$ and $y$; then

(by conic sections) $\qquad y^2=\dfrac{b^2}{a^2}(a^2-x^2)$

and subnormal $\mathrm{GN}=\dfrac{b^2x}{a^2}$

In triangle AGN, $y=n\sin.l$, and $\mathrm{GN}=n\cos.l$;

$\therefore\quad \dfrac{b^2}{a^2}(a^2-x^2)=n^2\sin.^2 l$, and $\dfrac{b^2x}{a^2}=n\cos.l$

$\therefore\quad x^2=\dfrac{n^2\,a^4}{b^4}\cos.^2 l$ in last equation.

Substituting this value of $x^2$ in former equation,

$$\frac{b^2}{a^2}\left(a^2-\frac{n^2\,a^4}{b^4}\cos.^2 l\right)=n^2\sin.^2 l$$

whence $\qquad n^2=\dfrac{b^4}{a^2\left(1-\dfrac{a^2-b^2}{a^2}\sin.^2 l\right)}$

$$\therefore \quad n=\frac{b^2}{a}\left(1-\frac{a^2-b^2}{a^2}\sin.^2 l\right)^{-\frac{1}{2}}$$

but $\quad c=a-b \quad \therefore \quad b=a-c$

$$\therefore \quad n=\frac{b^2}{a}\left(1-\frac{2ac-c^2}{a^2}\sin.^2 l\right)^{-\frac{1}{2}}$$

$$=\frac{b^2}{a}\left(1-\frac{2c}{a}\sin.^2 l\right)^{-\frac{1}{2}} \text{ nearly, since } \frac{c^2}{a^2} \text{ is small}$$

$$=\frac{b^2}{a}\left(1+\frac{c}{a}\sin.^2 l\right) \text{ neglecting terms involving higher powers of } \frac{c}{a}$$

Again, $\quad r=\dfrac{\left\{1+\left(\dfrac{dy}{dx}\right)^2\right\}^{\frac{3}{2}}}{-\dfrac{d^2y}{dx^2}}=\dfrac{\left\{b^2x^2+a^2(a^2-x^2)\right\}^{\frac{3}{2}}}{a^4b}$

and $\quad n^2=GN^2+AN^2$

$$=\frac{b^4x^2}{a^4}+\frac{b^2}{a^2}(a^2-x^2)=\frac{b^2}{a^4}\left\{b^2x^2+a^2(a^2-x^2)\right\}$$

$$\therefore \quad n^3=\frac{b^3}{a^6}\left\{b^2x^2+a^2(a^2-x^2)\right\}^{\frac{3}{2}}$$

$$\therefore \quad \frac{n^3a^6}{b^3}=\left\{b^2x^2+a^2(a^2-x^2)\right\}^{\frac{3}{2}}$$

Substituting this value of the numerator in $r$, we have

$$r=\frac{n^3a^2}{b^4}$$

$$\therefore \quad r=\frac{a^2}{b^4}\cdot\frac{b^6}{a^3}\left(1+\frac{c}{a}\sin.^2 l\right)^3$$

$$=\frac{b^2}{a}\left(1+\frac{3c}{a}\sin.^2 l\right) \quad \text{nearly}$$

$$=\frac{(a-c)^2}{a}\left(1+\frac{3c}{a}\sin.^2 l\right) \quad ,,$$

$$=\left(a-2c+\frac{c^2}{a}\right)\cdot\left(1+\frac{3c}{a}\sin.^2 l\right) \text{ nearly}$$

$$r=(a-2c)\cdot\left(1+\frac{3c}{a}\sin.^2 l\right) \text{ nearly}$$

$$=a+3c\sin.^2 l-2c \quad ,,$$

Similarly $r_1=a+3c\sin.^2 l_1-2c$

$$\therefore \quad r-r_1=3c\,(\sin.^2 l-\sin.^2 l_1)$$

$$\therefore \quad c=\frac{r-r_1}{3\,(\sin.^2 l-\sin.^2 l_1)}$$

$$=\frac{m\,(D-D_1)}{3(\sin.^2 l-\sin.^2 l_1)} \text{ (putting } m=57\cdot29577)$$

$$\text{or } c\text{ (in miles)}=\frac{m\,(D-D_1)}{3\times1760\times3}\operatorname{cosec}.(l+l_1)\operatorname{cosec}.(l-l_1)$$

Hence this rule to compute $c$.

Add together the constant log. $\overline{3}\cdot558367$ (the log. of $\frac{57\cdot29577}{3\times1760\times3}$), the log. of the difference of the lengths (in feet) of a degree in the two latitudes, and the log. cosecants of the sum and difference of the two latitudes: the natural number of the sum (rejecting 20 in the index) will be the value of $c$ (the difference between the semi-major and semi-minor axes) in miles.

EXAMPLES.

Find the value of $c$ from the measurements (2 and 9) of a degree in Russia and India (see Table).

| | | | |
|---|---|---|---|
| $l$ | $=58°\ 17'\ 37''$ | const. log. . . . . | $\overline{3}\cdot558367$ |
| $l_1$ | $=16\ \ 8\ 22$ | log. $(D-D_1)$ . . | $3\cdot366236$ |
| $l+l_1$ | $=74\ 25\ 59$ | log. cosec. $(l+l_1)$ | $0\cdot016230$ |
| $l-l_1$ | $=42\ \ 9\ 15$ | log. cosec. $(l-l_1)$ | $0\cdot173195$ |
| | | log. $c$ | $=1\cdot114028$ |
| D | $=365368$ | $\therefore \quad c$ | $=13\cdot00$ miles. |
| $D_1$ | $=363044$ | | |
| $D-D_1$ | $=\ \ 2324$ | | |

With the measurements in England and India (3 and 9), the value of $c = 12{\cdot}58$

Measurements 1 and 11 $c = 12{\cdot}83$

3 „ 10 $c = 12{\cdot}13$

2 „ 10 $c = \overline{12{\cdot}59}$

mean value of $c = 12{\cdot}64$ nearly.

The value of $c$ being found, we may compute $a$ by the formula,

$$r = a - 2c + 3c \sin.^2 l$$

whence

$$a = r + 2c - 3c \sin.^2 l$$

$$= m\mathrm{D} + (2 - 3 \sin.^2 l) \cdot \frac{m(\mathrm{D} - \mathrm{D}_1)}{3(\sin.^2 l - \sin.^2 l_1)}$$

$$= \frac{m}{3} \cdot \frac{2(\mathrm{D} - \mathrm{D}_1) + 3(\mathrm{D}_1 \sin.^2 l - \mathrm{D} \sin.^2 l_1)}{\sin.^2 l - \sin.^2 l_1}$$

$$= \frac{57{\cdot}29577}{9 \times 1760} (2\overline{\mathrm{D} - \mathrm{D}_1} + 3\overline{\mathrm{D}_1 \sin.^2 l - \mathrm{D} \sin.^2 l_1}) \operatorname{cosec.} \overline{l + l_1} \operatorname{cosec.} \overline{l - l_1}$$

Hence this rule to compute the semi-major axis $a$.

To the log. of the length of a degree ($\mathrm{D}_1$) in feet, in one latitude, add twice the log. sine of the other latitude. Again, to the log. of the length of a degree (D) in one latitude, add twice the log. sine of the other latitude (rejecting 20 from the index in each case.) Take the difference between the natural numbers of the resulting logarithms, and multiply by 3. Add thereto twice the difference between the lengths of a degree in each latitude, and take out the logarithm of the result. To this logarithm, add const. log. $\overline{3}{\cdot}558367$, and the log. cosecants of the sum and difference of the two latitudes. The sum (rejecting the tens in index) is the log. of the semi-major axis $a$, which find in the tables.

EXAMPLE.

Find the value of $a$ (the semi-major diameter of the earth) from the measurements (3 and 10) of a degree in England and India (see Table).

$l = 52° 35' 45''$
$l_1 = 12\ 32\ 21$
$l+l_1 = 65\ 8\ 6$
$l-l_1 = 40\ 3\ 24$

$D = 364971$
$D_1 = 363013$
$1958$
$2$
$2(D-D_1) = 3916$

| | |
|---|---|
| log. $D_1$ | 5·5599222 |
| log. sin. $l$ | 9·9000231 |
| log. sin. $l$ | 9·9000231 |
| | 5·3599684 |
| | 229070 |
| | 17203 |
| | 211867 |
| | 3 |
| | 635601 |
| $2(D-D_1)$ | 3916 |
| | 639517 |

| | |
|---|---|
| log. D | 5·5622584 |
| log. sin. $l_1$ | 9·3366737 |
| log. sin. $l_1$ | 9·3366737 |
| | 4·2356058 |
| | 17203 |
| const. log. . . . . | $\bar{3}$·5583670 |
| log. cosec. $(l+l_1)$ | 0·0422486 |
| log. cosec. $(l-l_1)$ | 0·1914211 |
| log. 639517 . . . | 5·8058521 |
| log. $a$ . . | 3·5978888 |

$\therefore$ semi-major diameter $a = 3961{\cdot}7$ miles.
Using Nos. 3 and 9 . . . . . $a = 3961{\cdot}8$
" 2 „ 10 . . . . . $a = 3962{\cdot}6$

Hence the mean value of $a = 3962$ miles, and of $c = 12{\cdot}64$ miles; $\therefore$ $b = 3962 - 12{\cdot}64 = 3949$ miles nearly, and $\frac{c}{a}$ (called the compression) $= \frac{12{\cdot}64}{3962} = \frac{1}{313{\cdot}4}$ nearly.

The distance CA of any point A on the surface of the earth from the center C will be useful hereafter in finding the correction of the moon's equatorial horizontal parallax. It may be computed as follows:

$$CA^2 = CN^2 + AN^2$$

and (conic sections) subnor. $GN = \frac{b^2}{a^2} CN$

$$\therefore \quad CN = \frac{a^2}{b^2} \cdot GN = \frac{a^2}{b^2} n \cos. l$$

$$= \frac{a^2}{b^2} \cdot \frac{b^2}{a} \left(1 + \frac{c}{a} \sin.^2 l\right) \cos. l \quad \ldots \text{ (p. 53)}$$

$$= a \cos. l \left(1 + \frac{c}{a} \sin.^2 l\right)$$

and $$AN = n \sin. l = \frac{b^2}{a} \left(1 + \frac{c}{a} \sin.^2 l\right) \sin. l$$

$$= \frac{(a-c)^2}{a} \left(1 + \frac{c}{a} \sin.^2 l\right) \sin. l$$

$$= \left(a - 2c + \frac{c^2}{a}\right) \cdot \left(1 + \frac{c}{a} \sin.^2 l\right) \sin. l$$

$$= (a - 2c) \cdot \left(1 + \frac{c}{a} \sin.^2 l\right) \sin. l \text{ nearly}$$

Substituting these values of CN and AN,

$$CA^2 = \left\{a^2 \cos.^2 l + (a-2c)^2 \sin.^2 l\right\} \cdot \left(1 + \frac{c}{a} \sin.^2 l\right)^2$$

$$= \left(a^2 \cos.^2 l + a^2 \sin.^2 l - 4ac \sin.^2 l + 4c^2 \sin.^2 l\right) \cdot \left(1 + \frac{2c}{a} \sin^2 l\right) \text{ nearly}$$

$$= a^2 + 2ac \sin.^2 l - 4ac \sin.^2 l \text{ nearly (neglecting other terms, which are small)}$$

$$= a^2 - 2ac \sin.^2 l$$

$$\therefore \quad \mathrm{CA} = a\left(1-\frac{2c}{a}\sin.^2 l\right)^{\frac{1}{2}} = a\left(1-\frac{c}{a}\sin.^2 l\right) \text{ nearly}$$

$$= a - c\sin.^2 l = 3962 - 12{\cdot}64\sin.^2 l.$$

Hence this rule to find the distance of any place AB from the center of the earth.

To the constant log. 1·101747 (the log. of 12·64) add twice the log. sine of the latitude of the place: the natural number of the result (rejecting the tens in index) subtracted from the semi-major diameter, 3962, is the distance required.

EXAMPLE.

Required the distance of a place in latitude 50° 48′ from the center of the earth.

| | |
|---|---|
| const. log. . . . | 1·101747 |
| log. sin. lat. . . | 9·889271 |
| ,, . . . . | 9·889271 |
| log. cor. . . . | 0·880289 |
| ∴ cor. . . | 7·6 miles. |

Hence distance from center = 3962 − 7·6 = 3954·4 miles.

The preceding formulæ will enable us to investigate an expression for computing the reduced latitude AC$q$ (or, as it is sometimes called, the latitude *on the sphere*, or central latitude) from the true latitude AG$q$, the latitude on the spheroid.

Let true latitude AG$q = l$

and reduced lat. AC$q = l_1$

then $l - l_1$ = the correction required.

$$\tan. l = \frac{\mathrm{AN}}{\mathrm{GN}} \qquad \tan. l_1 = \frac{\mathrm{AN}}{\mathrm{CN}}$$

$$\therefore \quad \frac{\tan. l}{\tan. l_1} = \frac{\mathrm{CN}}{\mathrm{GN}} = \frac{a^2}{b^2} \text{ (since } \mathrm{GN} = \frac{b^2}{a^2}\mathrm{CN}, \text{ p. 57)}$$

$$\therefore \text{ tan. } l_1 = \frac{b^2}{a^2} \text{ tan. } l = \frac{(a-c)^2}{a^2} \text{ tan. } l$$

$$= \left(1 - \frac{2c}{a}\right) \text{tan. } l \text{ nearly.}$$

$$\text{Now tan. } (l - l_1) = \frac{\text{tan. } l - \text{tan. } l_1}{1 + \text{tan. } l \,.\, \text{tan. } l_1}$$

$$= \frac{\text{tan. } l. \left(1 - 1 + \frac{2c}{a}\right)}{1 + \text{tan.}^2 l \left(1 - \frac{2c}{a}\right)}$$

$$= \frac{2c}{a} \cdot \frac{\text{tan. } l}{1 + \text{tan.}^2 l} = \frac{c}{a} \text{ sin. } 2l$$

$$= \frac{1}{313 \cdot 4} \text{ sin. } 2l$$

Since $l - l_1$ is only a few minutes, the circular measure $\left(\frac{\text{arc } (l - l_1)}{\text{rad.}}\right)$ may be substituted for tan. $(l - l_1)$;

$$\therefore \text{ arc } (l - l_1) \text{ in min.} = \frac{57^\circ \cdot 29577 \times 60}{313 \cdot 4} \text{ . sin. } 2l$$

$$= 11 \text{ sin. } 2l \text{ nearly.}$$

Hence this rule for finding the correction of the latitude for the spheroidal figure of the earth.

To the constant log. 1·041393 (the log. of 11) add the log. sine of twice the latitude (rejecting 10 in the index): the natural number of the result will be the correction required.

EXAMPLE.

The true latitude of a place being 54°, required the central or reduced latitude.

| | |
|---|---|
| const. log. . . . . | 1·041393 |
| sin. 108° . . . . | 9·978206 |
| log. cor. . . . . | 1·019599 |

$\therefore$ cor. $10'\cdot46 = 10'\ 26''$

and reduced latitude $= 54° - 10'\ 26'' = 53°\ 49'\ 34''$

# SOLUTIONS OF PROBLEMS.

## SECTION I.

### PROBLEMS IN SURVEYING, &c.

PROB. 1 (Fig. 17).

Let C*b* represent the height of tower = 216 feet
A*a* the height of spectator's eye = 5 feet
AB or *ab* the width of river.

Suppose AB parallel to horizontal line *ab*: join CA; then CAB the angle of elevation = 47° 56′, and CB = height of tower − 5 = 211 feet, are given, to find AB or *ab*, the width of river.

In right-angled triangle ACB, $\frac{AB}{BC} = \cot. CAB$ (Art. 5 *d*, p. 2*).

$$\therefore AB = BC \,.\, \cot. CAB$$

In logarithms,

$$\log. AB = \log. BC + \log. \cot. CAB - 10 \quad \text{(Rule 13, p. 30.)}$$
$$= \log. 211 + \log. \cot. 47° 56' - 10$$

*Calculation.*

| | |
|---|---|
| log. 211 . . . . . | 2·324282 } |
| log. cot. 47° 56′ . . . | 9·955453 } −10 |
| log. AB . . . . . | 2·279735 |

$\therefore$ AB = 190·4 feet, the width of river.

* The references are to Trigonometry, Part I.

PROB. 2 (Fig. 53).

(1.) In right-angled triangle ABD, given AD=100 feet, and angle B=36° 48′, to find BD.

(2.) In right-angled triangle ADC, given AD=100 feet, and angle C=54° 30′, to find DC.

Then BD+DC=BC, the line required.

*Calculation.*

In triangle ABD, $\frac{BD}{AD}=\cot. B$

or $BD=AD \cot. B$

$\therefore$ log. BD=log. AD+log. cot. B−10

=log. 100+log. cot. 36° 48′−10

log. 100 . . . . . 2·000000 } −10

log. cot. 36° 48′ . . . 10·126043 }

log. BD . . . . . 2·126043

$\therefore$ BD=133·7 feet.

In triangle ADC, $\frac{DC}{AD}=\cot. C$

or $DC=AD \cot. C$

$\therefore$ log. DC=log. AD+log. cot. C−10

=log. 100+log. cot. 54° 30′−10

log. 100 . . . . . 2·000000 } −10

log. cot. 54° 30′ . . 9·853268 }

log. DC . . . . . 1·853268

$\therefore$ DC= 71·3

and BD=133·7

$\therefore$ BD+DC=205·0 feet.

PROB. 3 (Fig. 17).

In right-angled triangle ABC, given AB=200, and A=67°, to find BC and AC, the other two sides.

(1.) *To find* BC.

$$\frac{BC}{AB} = \tan. A$$

or $BC = AB \tan. A$

$\therefore$ log. BC = log. AB + log. tan. A − 10

log. tan. A . . . 10·372148 } −10
log. AB . . 2·301030 }

log. BC . . . 2·673178

$\therefore$ BC = 471·1

(2.) *To find* AC.

$$\frac{AC}{AB} = \sec. A$$

or $AC = AB \sec. A$

$\therefore$ log. AC = log. AB + log. sec. A − 10

log. sec. A . . . 10·408122 } −10
log. AB . . . 2·301030 }

log. AC . . . 2·709152

$\therefore$ AC = 511·9

AB = 200 BC = 471·1 and AC = 511·9

$\therefore$ AB + BC + AC = 1183 feet = perimeter of triangle
= ·224 of a mile

Hence 4 : ·224 :: 60 : $x$ (time required)

log. ·224 . . . $\bar{1}$·350248
log. 60 . . . . 1·778151

1·128399

log. 4 . . . . 0·602060

log. $x$ . . . . . 0·526339

$\therefore$ $x$ = 3·36 min. or 3 min. 21·6 seconds.

PROB. 4 (Fig. 53).

In triangle BAC, AC=AB, and AD the perpendicular upon BC bisects angle BAC and side BC.

$\therefore$ DC=50 feet, and angle DAC=75°, are given, to find AC.

In triangle ADC, $\frac{AC}{DC}$ = cosec. DAC

or AC=DC . cosec. DAC

$\therefore$ log. AC=log. DC+log. cosec. DAC−10

| | | |
|---|---|---|
| log. DC . . . . . | 1·698970 | } −10 |
| log. cosec. DAC . . | 10·015056 | |
| log. AC . . . . . | 1·714026 | |

$\therefore$ AC=51·77 feet.

PROB. 5 (Fig. 18).

Let AC represent the broken piece = 39, A the point where it struck the ground, and AB the distance of its top from the base=15; representing the known quantities AC by $a$, and AB by $b$, and the unknown part BC left standing by $x$, we have, in right-angled triangle ABC, $x^2=a^2-b^2$

$=(a+b).(a-b)$ . . . (Part I. p. 55)

In logarithms,

$2\log. x=\log.(a+b)+\log.(a-b)$

$\therefore \log. x=\frac{1}{2}\{\log.(a+b)+\log.(a-b)\}$

| | | |
|---|---|---|
| $a=39$ | log. $a+b$ . . | 1·732394 |
| $b=15$ | log. $a-b$ . . | 1·380211 |
| $a+b=54$ | | 2)3·112605 |
| $a-b=24$ | log. $x$ . . . | 1·556302 |

$\therefore x=36$

$\therefore$ whole height of pole = 39+36=75 feet.

PROB. 6 (Fig. 19).

Let D and E be the windows; DC=EC the ladder $=36=a$, BD height of lower window $=18{\cdot}9=b$, and AE height of other window $=30{\cdot}7=c$.

Draw FD parallel to AB, then FDE is the angle of elevation of E above D.

Let this angle be denoted by $\theta$, BC by $x$, AC by $y$.

(1.) In triangle ACE $y^2=a^2-c^2=(a+c).(a-c)$

(2.) In triangle CBD $x^2=a^2-b^2=(a+b).(a-b)$

From these equations $x$ and $y$ may be found as in the last problem, viz. $x=30{\cdot}7$, and $y=18{\cdot}8$.

$$\tan.\theta=\frac{EF}{FD}=\frac{c-b}{x+y}$$

$\therefore$ log. tan. $\theta-10=$log. $(c-b)-$log. $(x+y)$

or log. tan. $\theta=10+$log. $(c-b)-$log. $(x+y)$

| | |
|---|---|
| $c=30{\cdot}7$ | log. $(c-b)+10$ . 11·071882 |
| $b=18{\cdot}9$ | log. $(x+y)$ . . 1·694605 |
| $c-b=11{\cdot}8$ | log. tan. $\theta$ . . 9·377277 |
| $x=30{\cdot}7$ | $\therefore\ \theta=13°\ 24'\ 30''$ |
| $y=18{\cdot}8$ | Breadth of street 49·5 feet |
| $x+y=49{\cdot}5$ | and angle of elevation 13° 24′ 30″ |

PROB. 7 (Fig. 17).

Ex. 1. Let BC represent the height of tower $=x$

AB the base measured $=a$

and angle BAC observed $=A$

In right-angled triangle BAC, $\frac{x}{a}=\tan.A$

or $x=a\tan.A$

$\therefore$ log. $x=$log. $a+$log. tan. A$-10$

| | | | |
|---|---|---|---|
| $a=50$ | log. $a$ . . | 1·698970 | $-10$ |
| A$=25°\ 17'$ | log. tan. A . | 9·674257 | |
| | log. $x$ . . | 1·373227 | |

whence height $x=23{\cdot}6$ yards.

Ex. 2. In this example, since BAC$=45°$

$\therefore$ BCA$=45°$ and BC$=$BA$=12\frac{1}{4}$ yards.

PROB. 8 (Fig. 20).

Ex. 1. Let C be the place of the ship, AB the base $=1000$ yards, A$=32°\ 10'$, and B$=83°\ 18'$.

In triangle ABC, the side AB (or $c$) and the two angles A and B are given, whence C may be found, and (by Rule III.) the side CA (or $b$).

$b : c$ :: sin. B : sin. C (by Rule III.)

| | | |
|---|---|---|
| A$=32°\ 10'$ | log. $c$ . . | 3·000000 |
| B$=83\ \ 18$ | log. sin. B . | 9·997024 |
| $115°\ 28'$ | | 12·997024 |
| 180 | log. sin. C . | 9·955609 |
| $\therefore$ C$=64°\ 32'$ | log. $b$ . . | 3·041415 |

$\therefore$ $b=1100{\cdot}1$ yards.

Ex. 2.

A+B$=\ 80°+\ 50°=130°$

$\therefore$ C$=180°-130°=\ 50°=$B

$\therefore$ sides opposite to B and C are equal, or AC$=2{\cdot}5$ miles.

PROB. 9 (Fig. 20).

The solution of this problem is in all respects similar to the last: thus,

To find AC . . AC : AB :: sin. B : sin. C

To find BC . . BC : AB :: sin. A : sin. C

whence AC$=9{\cdot}478$, and BC$=9{\cdot}52$ miles.

PROB. 10.

Let ABC be an isosceles triangle, A the vertical angle, draw AD perpendicular to BC (as fig. 53); AD bisects the vertical angle, and also the base BC; therefore, in the triangle ADC, DC=60 and angle DAC= 23° 30′ are given, to find AC.

$$\frac{AC}{DC}=\text{cosec. DAC}$$

or $$AC=DC \text{ cosec. DAC}$$

$$\therefore \log. AC=\log. DC+\log. \text{cosec. DAC}-10$$

whence AC or AB=150·5 feet.

PROB. 11.

(In fig. 17) draw BD perpendicular to the hypothenuse AC: it is required to find BD, having given AB=18 and BC=24.

(1.) Find angle A.

$$\tan. A=\frac{BC}{AB}$$

(2.) Given AB and A in right-angled triangle ADB, to find BD.

$$\frac{BD}{AB}=\sin. A$$

*Calculation.*

(1.) $\log. \tan. A-10=\log. BC-\log. AB$

$\log. \tan. A=\log. BC+10-\log. AB$

| | |
|---|---|
| $\log. BC+10$ . . . | 11·380211 |
| $\log. AB$ . . . . . | 1·255273 |
| $\log. \tan. A$ . . . . | 10·124938 |

$\therefore A=53° 7′ 45″$

(2.) $\log. BD = \log. AB + \log. \sin. A - 10$

| | |
|---|---|
| $\log. AB$ . . . . . | $1{\cdot}255273$ |
| $\log. \sin. A$ . . . . . | $9{\cdot}903085$ |
| $\log. BD$ . . . . . | $1{\cdot}158358$ |

$-10$

$\therefore BD = 14{\cdot}4$

PROB. 12 (Fig. 21).

Ex. 1. Let AB represent the height of tower, D and C the stations; then DC = 100 yards, ACB = 46° 15′, and ADB = 31° 20′.

(1.) In the triangle ADC are given DC = 100 yards, ADC = 31° 20′, and DCA = (180° − 46° 15′) = 133° 45′, to find AC.

(2.) In the right-angled triangle ACB are given AC and angle ACB, to find AB.

Ex. 2. In this example, CD = 300, ACB = 58°, and BDA = 32°.

*Calculation* Ex. 1.

$$\frac{AC}{} : DC :: \sin. ADC : \sin. DAC$$

| | | | |
|---|---|---|---|
| ADC = | 31° 20′ | $\log. DC$ . . | $2{\cdot}000000$ |
| DCA = | 133 45 | $\log. \sin. ADC$ | $9{\cdot}716017$ |
| | 165 5 | | $11{\cdot}716017$ |
| | 180 | $\log. \sin. DAC$ | $9{\cdot}410632$ |
| $\therefore$ DAC = | 14° 55′ | $\log. AC$ . . | $2{\cdot}305385$ |

$\therefore$ AC = 202

Then $\frac{AB}{AC} = \sin. ACB$, or $AB = AC \sin. ACB$

$\therefore \log. AB = \log. AC + \log. \sin. ACB - 10$

| | |
|---|---|
| $\log. AC$ . . | $2{\cdot}305385$ |
| $\log. \sin. ACB$ | $9{\cdot}858756$ |
| $\log. AB$ . . | $2{\cdot}164141$ |

$-10$

$\therefore AB = 145{\cdot}9$

PROB. 13 (Fig. 21).

The solution of this problem is similar to the last. Let AB represent the height of cloud, D and C the two ships; then DC=880 yards, ADC=35°, ACB=64° ∴ DCA=116°, and DAC=29°.

To find log. AC . . AC : DC :: sin. ADC : sin. DAC

| | |
|---|---|
| log. DC . . . . . | 2·944483 |
| log. sin. ADC . . . | 9·758591 |
| | 12·703074 |
| log. sin. DAC . . . | 9·685571 |
| log. AC . . . . . | 3·017503 |

To find AB . . . $\frac{AB}{AC}$ = sin. ACB

AB = AC sin. ACB

log. AB = log. AC + log. sin. ACB − 10

| | |
|---|---|
| log. AC . . . . . | 3·017503 |
| log. sin. ACB . . . | 9·953660 |
| log. AB . . . . . | 2·971163 |

∴ AB = 935·6 yards

and height above sea = 935·6 + 7 = 942·6 yards.

PROB. 14 (Fig. 21).

Let AB represent the tower, C and D the stations; then CB = 200, DB = 350, and angle ACB = 39°, are given, to find height AB and angle ADB.

To find AB . . . $\frac{AB}{CB}$ = tan. ACB

AB = CB tan. ACB

∴ log. AB = log. CB + log. tan. ACB − 10

log. CB . . 2·301030
log. tan. ACB 9·908369 } −10
log. AB . . 2·209399

∴ AB=162 nearly.

To find ADB . . . $\frac{AB}{DB}$=tan. ADB

∴ AB=DB tan. ADB

log. tan. ADB=log. AB+10−log. DB

log. AB+10 . . . 12·209399
log. DB . . . . . 2·544068
log. tan. ADB . . 9·665331

∴ ADB=24° 50′

Prob. 15 (Fig. 20).

Let B and C represent the two ships, A the spectator; then distances AC and AB are known, being 1142 × elapsed seconds, or AC=4568, AB=6852; hence in the triangle ABC are given two sides, and included angle A=48° 42′, to find the third side.

*Calculation* (by Rule V.).

constant log. . . 10·602060
log. hav. 48° 42′ . 9·230447
log. 6852 . . . . 3·835817
log. 4568 . . . . 3·659726
2)27·328050
13·664025 . . . . 13·664025
log. 2284 . . 3·358696
log. tan. . . 10·305329 log. sin. 9·952403
log. 3d side . . 3·711622

∴ 3d side=5147·9 feet.

Ex. 2. $AC = 1142 \times 10 = 11420$, and
$AB = 1142 \times 5 = 5710$, and angle $= 60°$.
$\therefore$ 3d side $= 9889{\cdot}8$ feet.

PROB. 16 (Fig. 21′).

Let AB represent the ship's mast, C the other ship's hull; then if BD is a horizontal line, DBC is the angle of depression, and is equal to BCA; then, in the right-angled triangle BAC are given $AB = 80$ and angle $C = 20°$, to find AC.

$$\frac{AC}{AB} = \text{cot. } C$$

or $AC = AB \text{ cot. } C$

$\therefore$ log. AC = log. AB + log. cot. C − 10

whence $AC = 219{\cdot}8$ feet.

Ex. 2. $AB = 143$, and angle $C = 35°$
$\therefore$ $AC = 204{\cdot}2$ feet.

PROB. 17.

Draw two parallel straight lines AB, CD, perpendicular to a horizontal line BD, to represent the two monuments; let $AB = 100$, $CD = 50$, and through the tops A, C draw a line cutting the horizontal line in E; then angle $E = 37°$; also since $AB = 2\,CD$ $\therefore$ $BE = 2\,DE$, and $DE = BD$.

To find DE or BD.

In the right-angled triangle CDE are given $CD = 50$ feet, and angle E 37°.

$$\frac{DE}{CD} = \text{cot. } E$$

$$DE = CD \text{ cot. } E$$

$\therefore$ log. DE = log. CD + log. cot. E − 10

whence $DE = 66{\cdot}4$ feet $= BD$, their distance apart.

PROB. 18 (Fig. 23).

Let AB represent the obelisk, C and D the two stations on the declivity.

(1.) In triangle ADC, given DC and all the angles, to find AC.

(2.) In triangle ACB, given two sides, BC, AC, and included angle ACB, to find the side AB, the height of the obelisk.

PROB. 19 (Fig. 24).

(1.) In triangle CAB, the angles CAB, ABC, and base AB, are given, to find AC.

(2.) In triangle DAB, the angles DAB, DBA, and base AB, are given, to find AD.

(3.) In triangle ADC, the two sides AC, AD, and included angle CAD (the difference between CAB and DAB), are given, to find CD, the required distance.

(1.) To find AC.

First find angle C.

| | |
|---|---|
| CAB . . . | 97° 56′ |
| ABC . . . | 42 22 |
| | 140 18 |
| | 180 |
| ∴ ACB= | 39° 42′ |

AC : AB :: sin. ABC : sin. ACB

| | |
|---|---|
| log. AB . . . | 2·826075 |
| log. sin. ABC . | 9·828578 |
| | 12·654653 |
| log. sin. ACB . | 9·805343 |
| log. AC . . . . | 2·849310 |

∴ AC=706·8

(2.) To find AD.

First find angle D.

| | |
|---|---|
| DAB . . . | 40° 16′ |
| ABD . . . | 113 29 |
| | 153 45 |
| | 180 |
| ∴ ADB= | 26° 15′ |

AD : AB :: sin. ABD : sin. ADB

| | |
|---|---|
| log. AB . . . | 2·826075 |
| log. sin. ABD . | 9·962398 |
| | 12·788473 |
| log. sin. ADB . | 9·645706 |
| log. AD . . . | 3·142767 |

∴ AD=1389·2

(3.) To find CD.

CAB . . . . . 97° 56′
DAB . . . . . 40 16
CAD=57° 40′

const. log. . . . . 10·602060
log. hav. A . . . . 9·366569
log. AD . . . . . 3·142767
log. AC . . . . . 2·849310
2)25·960706
12·980353 . . . . 12·980353
log. (AD−AC) . . 2·834039
log. tan. . . 10·146314 log. sin. 9·910551
log. CD 3·069802

∴ CD=1174·4 yards.

PROB. 20 (Fig. 24).

This useful problem is similar to the preceding.

PROB. 21.

Let ACB be the right-angled triangle resting upon its hypothenuse, C the right angle, and side $c=100$ feet.

Let A be the given angle=36° 40′.

$AC=x$, $CB=y$; and when the triangle is inclined at an angle of 60° to the horizon, let the distance of right angle C from the horizontal plane be denoted by $z$. In this case, $z$ will be the perpendicular in a right-angled triangle, of which $y$ is the hypothenuse, and angle at base=60°; then to find $z$.

(1.) In triangle ACB . . . . $x=c$ cos. A
(2.) In triangle ACD . . . . $y=x$ sin. A

∴ $y=c$ cos. A sin. A

(3.) The right-angled triangle ACB is inclined to the horizon at an angle $=60°$.

$\therefore\ z=y$ sin. $60°=c$ cos. A sin. A sin. $60°$, by substituting the value of $y$.

$\therefore$ log. $z=$log. $c+$log. cos. A $+$log. sin. A $+$log. sin. $60°-30$.

| | |
|---|---|
| log. $c$ . . . . . | 2·000000 |
| log. cos. A . . . | 9·904241 |
| log. sin. A . . . | 9·776090 |
| log. sin. $60°$ . . . | 9·937531 |
| log. $z=$ | 1·617862 |

$\therefore\ z=41·48$

PROB. 22 (Fig. 25).

(1.) In triangle DBC, given DC$=100$, D$=33°\ 45'$, and angle DBC$=$ACB$-$D$=51°-33°\ 45'=17°\ 15'$, to find log. BC.

(2.) In triangle CBE, given log. BC, angle BCE$=$ACB$-$ACE$=11°$, and angle CEB$=$A$+$ACE$=90°+40°=130°$, to find BE, the required height.

PROB. 23 (Fig. 26).

(1.) In triangle ACD, given CD and all the angles, to find AD.

(2.) In triangle BDE, given DE and all the angles, to find DB.

(3.) In triangle ADB, given AD, DB, and included angle ADB, to find the side AB, the required distance.

PROB. 24 (Fig. 27.)

(1.) In triangle ABD, given the three sides, to find the angle ABD, and thence its supplement ABO.

(2.) In triangle ACB, given the three sides, to find the angle CAB and its supplement BAO.

(3.) In triangle BAO, given BA and the angles, to find the sides BO and AO, the required distances.

PROB. 25 (Fig. 28).

(1.) In triangle BAO, the side AB and all the angles are known, to find log. AO.

(2.) In right-angled triangle OAC, the angle OAC and log. AO are known, to find AC and OC.

PROB. 26 (Fig. 29).

(1.) In oblique-angled triangle ACB, given side AC and all the angles, to find log. AB.

(2.) In right-angled triangle ABH, given angle BAH and log. hypothenuse, to find height BH.

PROB. 27 (Fig. 29).

Let BH represent the tower, A the place of the observer due south of tower, AC a straight line drawn due east, or at right angles to AH.

(1.) In triangle ABH are given, height of tower BH $=100$ feet and elevation BAH$=50°$, to find side AH$=$ 83·9.

(2.) In right-angled triangle ACH, right-angled at A, are given AH and AC, to find CH$=311·6$.

(3.) In right-angled triangle BCH, given BH and CH, to find angle of elevation BCH$=17°\ 47'\ 45''$.

PROB. 28 (Fig. 30).

Let A and B be the two observers, C the balloon; then angle of elevation CAD$=20°$

horizontal angle NAD$=45°$, the bearing of C from A

,, ABD$=11\frac{1}{4}°$ ,, C from B

(1.) In horizontal triangle ADB are given the side AB=4000 yards, ABD=11° 15′, and DAB=180°−45°, to find log. AD.

(2.) In vertical right-angled triangle ADC are given log. AD and CAD=20°, to find CD, the height of balloon.

PROB. 29 (Fig. 31).

(1.) In triangle BCD, given DC=18, the angle BCD=90°−40°, and angle BDC=90°+37° 30′, to find log. CB.

(2.) In right-angled triangle ABC, given log. CB and ACB=40°, to find height AB and distance AC of steeple.

PROB. 30 (Fig. 32).

(1.) In triangle BRS, given BR=54 feet, angle RBS =90°=4° 52′, and angle BSR=4° 52′−4° 2′, to find log. RS.

(2.) In right-angled triangle SAR, given log. RS, and ASR=NRS=4° 2′, to find AR the height and AS the distance.

PROB. 31 (Fig. 33).

(1.) Let CD be the breadth of the river; then in triangle ABC, given AB=500, and all the angles, to find log. BC.

(2.) In right-angled triangle DBC, given log. BC and angle B=79° 12′, to find perpendicular CD, the breadth of river.

Or thus,

In triangle ABC . . $\frac{BC}{AB}=\frac{\sin. A}{\sin. C}=\frac{\sin. A}{\sin. (A+B)}$

$\therefore$ BC=AB sin. A cosec. (A+B)

In triangle DBC, DC=BC sin. B

=AB sin.A sin.B cosec.(A+B)

PROB. 32 (Fig. 34).

Let AB be the sloping base, AD, BF horizontal lines in the vertical plane ACD, and CD the height of the hill; then the angle CBF=46° 15′, CAD=40° 10′, EBF=EAD=10° 5′.

$$\therefore \text{CBE}=\text{CBF}-\text{EBF}=36^\circ\ 10'$$

$$\text{CAE}=\text{CAD}-\text{EAD}=30^\circ\ 5'$$

$$\therefore \text{ACB}=6^\circ\ 5'$$

(1.) In triangle ACB, find log. AC.

(2.) In triangle ACD, find CD=1221, and adding height of eye 5 feet, the height of hill=1226 feet.

PROB. 33 (Fig. 35).

Let A be an object which we may suppose on the top of a hill, and B and C two stations on its sloping side. Conceive a horizontal plane to pass through the lowest, C, and let A*a* B*b* be perpendiculars on that plane, meeting it in *a* and *b*; join the points A, B, C in the *oblique* plane and the points *a*, *b*, C in the *horizontal* plane, and draw BD perpendicular to A*a*; then A*a*C, B*b*C will be right-angled triangles, and BD*ab* a parallelogram.

There are given, BC the distance between the stations=220 yards, horizontal angle *b*C*a*=62° 33′ subtended by A and B at C, horizontal angle C*ba*=70° 15′ subtended by A and C at B, and the vertical angles AC*a*, BC*b* the elevations of the object A and upper station B, to find the horizontal distances *ba*, C*a*, and the heights A*a*, AD.

(1.) In the right-angled triangle B*b*C, the hypothenuse BC=220 and angle BC*b*=8° 32′.

$\therefore$ perpendicular B*b*=32·6, and base C*b*=217·6.

(2.) In the horizontal triangle $ab$C, the side C$b$=217·6, the angles $ab$C=70° 15′, and $a$C$b$=62° 33′.

$\therefore$ $ab$=263·1, and $a$C=279·1.

(3.) In the right-angled triangle A$a$C, the side $a$C=279·1, and the acute angle AC$a$=32° 12′.

$\therefore$ A$a$=175·7.

(4.) And since D$a$=B$b$=32·6,

$\therefore$ AD=A$a$−B$b$=143·1.

PROB. 34 (Fig. 35).

(1.) In right-angled triangle BC$b$, BC=220 yards and BC$b$=8° 32′.

$\therefore$ B$b$=D$a$=32·6, and C$b$=217·6.

(2.) In triangle ABC, the side BC=220 and the three angles are known.

$\therefore$ log. AC=2·518244, and AB=299·5.

(3.) In right-angled triangle AC$a$, log. AC=2·518244 and angle AC$a$=32° 12′.

$\therefore$ A$a$=175·7, and C$a$=279·1.

(4.) In right-angled triangle ABD, AD=A$a$−B$b$=175·7−32·6=143·1, and AB=299·5.

$\therefore$ $ba$=BD=263·1 yards.

PROB. 35 (Fig. 36).

(1.) In the triangle CAD, CD=549·4 yards, angle C=57°, and CDA=14°.

$\therefore$ CAD=109°, and AD=487·27.

(2.) In triangle ABD, AD=487·27 and the three angles are known.

$\therefore$ AB=349·52, and BD=498·7.

PROB. 36 (Fig. 37).

(1.) In triangle AFC, given CA=3000, C=80° 10′, and CFA=25° 8′, to find FA=6959.

(2 ) In triangle ABF, given FA=6959, AB=2000, and included angle FAB=34° 10′; hence FB=5422, the distance required.

PROB. 37 (Fig. 38).

Describe a circle passing through the three points A, B, D, and join AE, EB. Then (by Geometry) since angles in the same segment are equal,

∴ EAB=EDB=15° 30′, and EBA=ADE=12° 15′.

(1.) In triangle AEB, the side AB=5·35, and angle EAB=15° 30′, and EBA=12° 15′.

∴ AE=2·438.

(2.) In triangle ABC, the three sides are given, to find CAB=35° 24′, and CAE (=CAB−EAB)=19° 54′.

(3.) In triangle CAE, the side AC=2·8, AE=2·438, and angle CAE=19° 54′.

∴ angle ACE=58° 33′.

(4.) In triangle ACD, the side AC=2·8, ADC=12° 15′, and ACD=58° 33′.

∴ AD=11·26, CD=12·46, and CAD=109° 12′.

Lastly. In triangle ABD, ADB = 27° 45′, BAD (=CAD−CAB)=73° 48′, and AB=5·35.

∴ BD=11·03.

PROB. 38 (Fig. 39).

Describe a circle passing through points A, B, D, as in last problem; then angle ADC=128° 40′ ∴ ADE=51° 20′; join AE, EB; then EAB=EDB=91° 20′−51° 20′=40°, and EBA=ADE=51° 20′.

(1.) In triangle AEB, given side AB=267, angle EAB=40°, and BEA=88° 40′, to find BE=171·6.

(2.) In triangle ABC, the three sides are given to find ABC=92° 26′; hence the angle EBC=143° 46′.

(3.) In triangle CBE are given two sides and included angle, to find BCD=16° 16′ 30″; whence DBC =23° 43′ 30″.

(4.) In triangle BDC are given BC and all the angles, to find BD=91 and DC=131, the two distances required.

(5.) In triangle ADB are given AB=267, ABD= 68° 42′ 30″ (=ABC−DBC), and ADB=91° 20′, to find AD=249, the third distance required.*

PROB. 39 (Fig. 40).

Let A be the place of the ship when the headland C bears due north of her; join AC, and draw AB at right angles to AC; let B be the place of the ship when the headland bears north-west; draw BN parallel to AN, to represent the direction of the meridian, or north and south line; then CBN (the bearing of C from B)= 4 points, or 45°.

∴ ∠ ABC=4 points, or 45°, and CAB is a right angle.

In right-angled triangle BAC are given AB=10 and angle BCA=45°; hence AC and BC may be found.

PROB. 40 (Fig. 41).

In triangle ACB, given angle C the course of the ship=4 points or 45°, the angle A=45° (since B bears south-east from A), and side CB=14·5, to find the sides AC and AB.

PROB. 41 (Fig. 42).

In triangle ACB, the bearing of B from A is east, and of C from A is N.N.E. ∴ angle CAB=6 points= 67° 30′; also angle ABC=4 points, and side BC=10; hence AB, the distance of boat from ship, can be found.

* The answers at p. 12 are incorrect.

PROB. 42 (Fig. 43).

In triangle ACB, the bearing of C from A is N.E. b. N. and of B from A is E. b. N. ∴ angle CAB (the difference between E. b. N. and N.E. b. N.) = 4 points, or 45°.

Again, the bearing of A from B is W. b. S., and of C from B is W.N.W. ∴ angle ABC = 3 points, and side AB = 15 miles; hence the other sides may be found.

PROB. 43.

In any triangle ABC, let AB = 50; make the angle A = 7 points, and angle B = 1 point; then if B is due south of A, and C is to the westward of AB, the bearing of C from A is W. b. S., and from B, N. b. W.

In the triangle ABC are given the side AB and all the angles; hence the sides AC and CB may be found.

PROB. 44 (Fig. 44).

(1.) In triangle CAB, given AB = 23 miles, CAB = $10\frac{1}{2}$ points, and CBA = $3\frac{1}{2}$ points ∴ CB = 53·01.

(2.) In triangle ABH, given AB = 23 miles, ABH = 8 points, and HAB = 4 points ∴ AHB = 4 points, and BH = AB = 23 miles.

(3.) In triangle CBH, given CB, BH, and included angle CBH = $4\frac{1}{2}$ points, to find CH = 42·33, the distance of cape from headland; and BCH = $2\frac{1}{4}$ points. Hence C bears from H $2\frac{1}{4}$ points to the left, *i.e.* $2\frac{1}{4}$ points left of W.N.W. or W.$\frac{1}{4}$S.

PROB. 45.

This problem is similar to the last, the position and bearing of the bodies only being different. Make a figure to suit the case (roughly or by scale) by drawing

from a point A the line AC W.N.W., the line AB N.W., and AH due north. Let AB = 6, and through B draw BC W.b.S., and BH N.E., and join CH.

(1.) In triangle CAB, find CB = 4·13.

(2.) In triangle ABH, find BH = 6.

(3.) Lastly, in triangle CBH are given two sides, and included angle CBH = 13 points, to find CH = 9·71 miles, and angle CHB = 13° 40′.

Hence C bears from B 13° 40′ to the right, and therefore bearing of C from H is S.(45° + 13° 40′)W., or S. 58° 40′ W.

PROB. 46 (Fig. 46).

Let A and C represent the stations on deck, H the object on shore; then angle A = 3 points (the bearing of H from C), and angle H = 2 points (the difference of bearings at the two stations). In the triangle, all the angles are given, and also the side AC = 100; hence the distance AH and CH may be found.

PROB. 47 (Fig. 45).

Let A be the required position of the ship; then in triangle ABC, the bearing of C from B is E. N. E., and A from B is due south ∴ the angle CBA = 10 points. We have ∴ given, BC = 960, AB = 2000, and included angle ABC = 10 points, to find the angle BAC = 20° 32′; hence the bearing of church is N. 20° 32′ E.

PROB. 48 (Fig. 46).

Let A be the required position of a ship; then in triangle ACH, bearing of C from H is W. $\frac{1}{4}$S. and of A from H is S. b. E. $\frac{1}{2}$E. ∴ angle CHA = $9\frac{1}{4}$ points. In triangle ACH we have given, the two sides CH = 4·23,

and AH = 2·3, and included angle CHA, to find angle HAC = 4½ points; hence the bearing of C is 4½ points to the left of H, that is, to the left of N. b. W. ½W., and the required bearing is ∴ W. N. W.

PROB. 49 (Fig. 47).

(1.) In horizontal triangle ADS, given AS = 2640, SD = 800, and angle ASD = 3 points = 33° 45′, to find AD = 2024·2.

(2.) In right-angled triangle ABD, AD = 2024·2, and ADB = 5° 25′, to find BA the height = 192 yards.

PROB. 50 (Fig. 48).

In triangle ABC, the angle ACB (the difference of bearings of A and B from C) is 3 points, the bearing of A from B is due west, and of C from B is S. W. b. W. ∴ angle ABC = 3 points, and side AC = 10 miles; hence BC may be found.

PROB. 51 (Fig. 49).

From D, the highest part of muzzle, draw DG parallel to axis AE of piece, then angle BDG = BCA, the angle required.

Through B draw a line perpendicular to axis, cutting DG in H, and AE in a point which mark K; then in right-angled triangle BDH are given BD = 77·25 inches, and BH (= BK − DE) = 6·22 − 4·92 = 1·3 inches, to find angle BDG.

PROB. 52 (Fig. 50).

(1.) In triangle AED . . . $\cot. AED = \frac{ED}{AD} = \frac{50}{175}$

∴ AED = 74° 3′ 15″, and AEB = 148° 6′ 30″ = angle of the tenaille.

(2.) In triangle BAK are given AB=350, BK=100, and included angle ABK(=90°−74°3′ 15″)=15°56′45″, to find AK=255·33=AG, the line of defence.

(3.) Since the angle of a regular pentagon=108°= HAH$_1$+2 BAH $\therefore$ $\frac{1}{2}$ HAH$_1$+BAH=54°, or $\frac{1}{2}$ HAH$_1$= 54°−15°56′45″=38°3′15″ $\therefore$ HAH$_1$=76°6′30″, the flanked angle.

(4.) To find curtain FG. In triangle AED, AE= $\sqrt{AD^2+DE^2}=\sqrt{175^2+50^2}=182$; hence GE or EF =AG−AE=255·33−182=73·33.

Then, in isosceles triangle GEF, are given two sides, GE and EF, and included angle GEF=148°6′30″, to find side GF=141·02, the curtain.

(5.) In triangle FEH are given EH=82, EF=73·33, and included angle HEF=31°53′30″ (supplement of 148°6′30″), to find angle EHF=63°; hence its supplement AHF=117°0′, the angle of the shoulder.

(6.) In triangle FEH are given EH and all the angles to find FH=43·5, the length of flank.

(7.) And flanked angle GFH=GFE+EFH=GFE +(AHF−HEF)=15°56′45″+85°6′30″=101°3′15″.

## Prob. 53 *a*.

Note.—The problems marked with the letter *a* require, in general, a knowledge of analytical trigonometry.

In fig. 1, suppose DB=6 times AB, and the angle ACB= double the angle D.

(1.) In triangle ADB . . . $\tan D=\frac{AB}{DB}=\frac{AB}{6.AB}=\frac{1}{6}$

$\therefore$ log. tan. D−10=log. 1−log. 6=−log. 6

og. tan. D=10−log. 6=9·221849

$\therefore$ D $= 9° 28'$

$\therefore$ angle ACB $= 18° 56'$.

(2.) To find the distance of C from tower.

In triangle ACB . . . . $\frac{CB}{AB} = \text{cot. ACB}$

$\therefore$ CB $=$ AB . cot. ACB.

We may either assume AB $= 1$, and thus get the value of CB in terms of AB, or find the natural cotangent of ACB.

log. cot. ACB $=$ tab. log. cot. ACB $- 10$ [Rule 13, p. 30.

$= 10{\cdot}464672 - 10 = {\cdot}464672$

$\therefore$ nat. cot. ACB $= 2{\cdot}9$

hence CB $= 2{\cdot}9 \times$ height AB.

PROB. 54 (Fig. 51).

Let AC represent the part standing the first time.

AE .. .. .. .. .. .. second ..

DC .. .. .. .. broken .. first ..

and EF .. .. .. .. .. .. second ..

then AD $= 20$, and AF $= 30$

let AE $= x$ $\therefore$ AC $= x+5$

CD $= y$ $\therefore$ EF $= y+5$

and $x+y+5 =$ height of may-pole.

In triangle FAE . . . . $x^2+30^2 = (y+5)^2$ . . . (1).

In triangle ACD . . . . $(x+5)^2+20^2 = y^2$ . . . (2).

By (1) $x^2+900 = y^2+10y+25$.

By (2) $x^2+10x+425 = y^2$

subtracting (1) from (2), $10x-475 = -10y-25$

or $10(x+y) = 450$

$\therefore$ $x+y = 45$

and $x+y+5 = 50$, the height required.

PROB. 55 *a* (Fig. 52).

Let $x$ = BD, the horizontal distance of A from B
$y$ = CD .. .. .. .. .. .. .. C
$z$ = AD, the height of hill.
$\alpha$ = elevation of A at B = 20°
$\beta$ = horizontal angle BCD, or the bearing of A from C = 67° 30′
and $\theta$ = ACD, the angle of elevation of A from C.

(1.) In right-angled triangle BCD . . . $x = y \sin. \beta$
(2.) In vertical triangle ADB . . . . . $z = x \tan. \alpha$
(3.) In vertical triangle ADC . . . . . $z = y \tan. \theta$

$\therefore$ $y \tan. \theta = x \tan. \alpha$ . . . from (2) and (3)
$= y \sin. \beta \tan. \alpha$ . . . from (1)

$\therefore$ $\tan. \theta = \sin. \beta \tan. \alpha$

hence $\theta = 18° 36'$.

PROB. 56 *a* (Fig. 53).

(1.) In the triangle ABC the three sides are given, to find one of the angles at the base, as B, and thence, in the right-angled triangle ADB, the perpendicular AD is easily found.

Or thus (without using logarithms):

Let AD = $x$; and segment BD = $y$; $a$, $b$, $c$ the three sides of the triangle; $a = 40$, $b = 30$, $c = 20$.

In right-angled triangle ABD . . . . . $x^2 + y^2 = c^2$ . . (1)
.. .. .. .. .. ADC . . $x^2 + (a-y)^2 = b^2$ . . (2)

Subtract (1) from (2), we have $(a-y)^2 - y^2 = b^2 - c^2$

$$\text{whence } y = \frac{a^2 + c^2 - b^2}{2a} = 13{\cdot}75$$

and $x = \sqrt{c^2 - y^2} = 14{\cdot}523$, the perpendicular required.

PROB. 57 (Fig. 54).

(1.) In triangle ABC are given all the angles and side AB, to find AC.

(2.) In right-angled triangle ABC are given AC and angle DAC, to find DC.

Or thus:

Let DAC $= \alpha = 23° 50' 15''$
CAB $= \beta = 93\ \ 4\ 20$
ABC $= \gamma = 54\ 28\ 30$
side AB $= a = 416$
AC $= x$, and DC $= y$.

(1.) In ABC . . . $\frac{x}{a} = \frac{\sin.\gamma}{\sin.\text{ACB}} = \frac{\sin.\gamma}{\sin.(\beta+\gamma)}$

or $x = a \sin.\gamma \operatorname{cosec}.(\beta+\gamma)$

(2.) In ADC . . . $y = x \tan.\alpha$.

$= a \sin.\gamma \operatorname{cosec}.(\beta+\gamma) \tan.\alpha.$

PROB. 58 *a* (Fig. 52).

Let distance BD of first station $= x$
.. .. .. CD of second .. $= y$
height AD of tower $= z$

angle ABD $= \alpha = 23° 18'$, angle ACD $= \beta = 21° 16'$, and side BC $= 300$.

In triangle ABD . . . . $z = x \tan.\alpha$ . . . . . (1)

In triangle ACD . . . . $z = y \tan.\beta$ . . . . . (2)

$\therefore\ x \tan.\alpha = y \tan.\beta$, and $\frac{x}{y} = \frac{\tan.\beta}{\tan.\alpha}$

but in triangle BCD . . . . $\frac{x}{y} = \cos.\text{BDC}$

$\therefore\ \cos.\text{BDC} = \frac{\tan.\beta}{\tan.\alpha} = \tan.\beta \cot.\alpha.$

whence BDC = 25° 20′ 45″.

Then in triangle BDC . . BD = BC cot. BDC = 633·4,

and AD = BD tan. $\alpha$ = 272·7.

### Prob. 59.

In figure 20, let AC represent the pole, inclined at an angle A = 80°, B the place of spectator; then AB = 100 feet, and the angle B = 54°. In triangle ABC are given side AB and all the angles, to find AC, the length of pole.

### Prob 60 *a* (Fig. 55).

Let AE (height of man) = $a = 6$ feet

AB (height of column) = $a+b = 200$ feet

and BC (height of statue) = $c = 50$ feet.

Let $\theta$ = ADE or BDC, the angle subtended at D, the place of spectator, by $a$ and $c$.

$x$ = BDE, and $y$ = AD, the width of the river.

(1.) In triangle AED . . . $\frac{a}{y} = \tan.\ \theta$

(2.) .. .. ABD . . . $\frac{a+b}{y} = \tan.\ (\theta+x)$

(3.) .. .. ACD . . . $\frac{a+b+c}{y} = \tan.\ (2\theta+x) =$

$\tan.\ (\overline{\theta+x}+\theta) = \frac{\tan.\ (\theta+x)+\tan.\ \theta}{1-\tan.\ (\theta+x)\tan.\ \theta}$ (Trig. Part II. p. 35.)

$$= \frac{\frac{a+b}{y}+\frac{a}{y}}{1-\frac{a+b}{y}\cdot\frac{a}{y}} = \frac{(2a+b)y}{y^2-(a+b)a} \text{ from (1) and (2).}$$

$$\therefore \frac{a+b+c}{y^2} = \frac{2a+b}{y^2-(a+b)a}$$

Substituting in this equation the values of $a$, $b$, $c$, we have

$$\frac{250}{y^2}=\frac{206}{y^2-1200}$$

or $\frac{206}{250}=\frac{y^2-1200}{y^2}=1-\frac{1200}{y^2}$; wherefore

$\frac{1200}{y^2}=1-\frac{206}{250}=\frac{22}{125}$ and $y=82{\cdot}5=$ the width of river.

PROB. 61 *a* (Fig. 24).

Let AB $=x$, AC $=y$, AD $=z$;

(1.) In triangle CAB, $y : x ::$ sin. ABC : sin. ACB

$$\therefore\ y=\frac{x \text{ sin. ABC}}{\text{sin. ACB}}=x \text{ sin. ABC cosec. ACB}=\cdot 8465x$$

(2.) In triangle ABD, $z : x ::$ sin. ABD : sin. ADB

$$\therefore\ z=\frac{x \text{ sin. ABD}}{\text{sin. ADB}}=x \text{ sin. ABD cosec. ADB}=1{\cdot}1852x$$

(3.) In triangle CAD, let CD or $6594=a$, $1{\cdot}1852=b$, $\cdot 8465=c$; then $z=bx$, and $y=cx$, and angle CAD $=85°\,46'=$ A. To find $x$ or AB.

In triangle ACD, $a=\frac{\sqrt{4yz \text{ hav. A}}}{\text{sin. }\theta}$

where tan. $\theta=\frac{\sqrt{4yz \text{ hav. A}}}{z-y}$ (Trig. Part II. p. 53.)

substituting $cx$ and $bx$, the values of $y$ and $z$ found above,

we have tan. $\theta=\frac{\sqrt{4\,cx\,bx \text{ hav. A}}}{bx-cx}$

$$=\frac{\sqrt{4bc \text{ hav. A}}}{b-c}$$

from which $\theta$ may be found.

and $a=\frac{\sqrt{4\,cx\,bx \text{ hav. A}}}{\text{sin. }\theta}$

whence $x = \dfrac{a \sin. A}{\sqrt{bc \text{ hav. } A}}$

Or thus (without haversines):

Find the angles of triangle ACD by Rule (IV.), and thence $y$ and $x$: thus

$$bx + cx : bx - cx :: \tan. \tfrac{1}{2}(ACD + ADC) : \tan. \tfrac{1}{2}(ACD - ADC)$$

or $$b + c : b - c :: \tan. \tfrac{1}{2}(ACD + ADC) : \tan. \tfrac{1}{2}(ACD - ADC)$$

whence the angles may be found, and thence $y$ and $x$.

PROB. 62 (Fig. 56).

Let BC (height of tower) $= x$; CD (the flagstaff) $= 4$ yards $= a$; AB (distance of observer) $= 100$ yards $= b$; $\alpha =$ DAC $= 48' 20''$, and $\varphi =$ angle BAC subtended by tower.

(1.) In triangle BAC, $\dfrac{x}{b} = \tan. \varphi$.

(2.) In triangle BAD, $\dfrac{x+a}{b} = \tan. (\varphi + \alpha) =$

$$\frac{\tan. \varphi + \tan. \alpha}{1 - \tan. \varphi \tan. \alpha} = \frac{x + b \tan. \alpha}{b - x \tan. \alpha} \ldots \text{ from (1).}$$

By solving the quadratic equation $\dfrac{x+a}{b} = \dfrac{x + b \tan. \alpha}{b - x \tan. \alpha}$

we have, $x = -\dfrac{a}{2} \pm \sqrt{\left\{\dfrac{a^2}{4} + \dfrac{ab - b^2 \tan. \alpha}{\tan. \alpha}\right\}}$ from which $x$ may be found as follows:

*Calculation.*

log. tan. $\alpha$ = tabular log. tan. $\alpha - 10$ . . . (Trig. Part I. Rule XIII.)
$= 8{\cdot}148001 - 10$
$= \overline{2}{\cdot}148001$
its Nat. No. or natural tang. $\alpha = {\cdot}01406$

$$\therefore\ x = -2 \pm \sqrt{\left\{4 + \frac{400 - 10000 \times \cdot 01406}{\cdot 01406}\right\}}$$

$$= 133 \cdot 8 \text{ yards} = 401 \cdot 4 \text{ feet.}$$

(See also Calculation of Prob. 80.)

Or thus:

In problems of this kind, where one of the angles concerned is small (only a few minutes), the *circular measure* of the angle may be sometimes used with advantage (see Trig. Part II. p. 110), thus:

Let CD (height of flagstaff) $= a = 4$ yards

AB (distance of spectator) $= b = 100$ yards

BC (height of tower) $= x$

angle DAC $= \varphi$.

With center A and distance AC describe the arc CE, cutting AD in E; then triangle ECD, being small, may be considered as a plane triangle, right-angled at E, and equiangular to the triangle BAC. Now circular measure

$$\text{of angle DAC} = \frac{\text{arc}}{\text{rad.}} = \frac{48' 20''}{57^\circ \cdot 29577} = \frac{\text{EC}}{\text{AC}}$$

but $\text{EC} = a \cos. \text{ECD} = a \cos. \text{BAC}$

and $\text{AC} = b \sec. \text{BAC}$

$$\therefore\ \frac{\text{EC}}{\text{AC}} = \frac{a \cos. \text{BAC}}{b \sec. \text{BAC}} = \frac{a}{b} \cos.^2 \text{BAC}$$

$$\therefore\ \cos.^2 \text{BAC} = \frac{100 \times 48' 20''}{4 \times 57^\circ \cdot 29577}$$

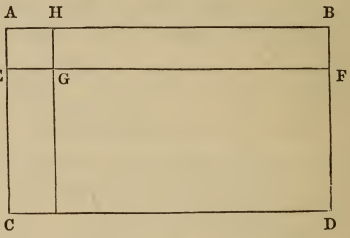
$$= \frac{25 \times 0^\circ \cdot 80555}{57^\circ \cdot 29577}$$

$$= \frac{25 \times 80555}{5729577}$$

$\therefore$ BAC $= 53^\circ\ 38'\ 30''$, and hence $x = 407 \cdot 5$.

## Prob. 63.

Since the area of a parallelogram is equal to the product of its base by its altitude,* and the area of triangle $= \frac{1}{2}$ area of a parallelogram with same base and altitude




* Proof that area AD = AB.AC.

(1.) Let each of the sides AB, AC of the rectangular parallelogram AD contain AH or AE, the linear unit, an exact number of times; that is, let AB and AC be *commensurable* with AH.

Let the linear unit AH or AE $= x$, and the square AG = superficial unit.

Let AB contain $a$ linear units $= ax$

AC . . . . $b$ . . . . . $= bx$

Then it may be shewn that the *area* or number of superficial units in AD $= ab$, or area AD $= ab$ times AG.

For (Eucl. VI. 1) $\frac{AF}{AG} = \frac{AB}{AH} = \frac{ax}{x} = a$

$$\frac{AD}{AF} = \frac{AC}{AE} = \frac{bx}{x} = b$$

$\therefore$ multiplying $\frac{AD}{AG} = ab$

or AD $= ab$ times AG.

(2.) If the sides AB, AC be *incommensurable* with AH, another unit may be found which is commensurable with certain lines that approach *as near as we please to* AB *and* AC, and therefore the product of the units in such lines will represent the area of a rectangle differing from the area AD by less than any assignable quantity; that is, we may in this case also express the area of AD by the product of the number of units in AB and AC.

$\therefore$ area triangle $= \frac{1}{2}$ base $\times$ alt. $= \frac{1}{2} \times 40 \times 30 = 600$ square feet $= 66\frac{2}{3}$ square yards.

PROB. 64 *a*.

Let $x =$ base of triangle cut off
$y =$ perpendicular
then (by last proof) $xy = 48$ . . . . (1)
and $x : y :: 12 : 16$
$\therefore \quad 4x = 3y$ . . . . . (2)
from equations (1) and (2) $x = 6$, $y = 8$
whence hypothenuse $= \sqrt{8^2+6^2} = 10$.

PROB. 65 *a*.

Let $x =$ side of equilateral triangle ABC.
Now area $= \sqrt{f.(f-a).(f-b).(f-c)}$
where $f = \frac{1}{2}(a+b+c)$ . . . (See Trig. Part II. p. 53.)
$$= \frac{3x}{2} \quad \text{(since } a = b = c = x\text{)}$$
$$\therefore \text{ area} = \sqrt{\frac{3x}{2} \cdot \frac{x}{2} \cdot \frac{x}{2} \cdot \frac{x}{2}} = \frac{\sqrt{3}}{4} x^2$$
$$\text{or } 180 = \frac{\sqrt{3}}{4} x^2, \text{ whence } x = 20{\cdot}389.$$

PROB. 66 *a*.

In any triangle ABC (fig. 20) are given the base AB and the angles A and B, to find the area.

Find one of the sides AC (Rule III.) and thence the area, having two sides, AC, AB, and included angle A, given. (Rule VI.)

Or thus:

$$\frac{AC}{AB} = \frac{\sin. B}{\sin. C} = \frac{\sin. B}{\sin. (A+B)}$$

$$\therefore \quad AC = AB \sin. B \operatorname{cosec}.(A+B)$$

and $2\,\text{area} = AB\,AC \sin. A = AB^2 \sin. A \sin. B \operatorname{cosec}.(A+B)$

PROB. 67 *a* (Fig. 20).

In triangle ABC, let $a=5$, $b=3$, and $x=$ third side,

then $\text{area}^2 = 36 = f.(f-a).(f-b).(f-x)$

where $f = \frac{1}{2}(a+b+x) = \frac{1}{2}(8+x)$

$$\therefore \quad 36 = \left(\frac{8+x}{2}\right).\left(\frac{8+x}{2}-5\right).\left(\frac{8+x}{2}-3\right).\left(\frac{8+x}{2}-x\right)$$

$$= \frac{(8+x).(x-2).(2+x).(8-x)}{16} = \frac{(64-x^2).(x^2-4)}{16}$$

whence, solving this quadratic, $x = 4$ or $\sqrt{52}$.

PROB. 68 *a* (Fig. 57).

By the question, $2.AEF = ABC$.

Let altitude AD of triangle ABC $= a = 10$

base $BC = x$

$EF = y$, base of triangle AEF

and $z =$ its altitude

then area $AEF = \frac{1}{2}$ ABC

$$\therefore \quad \frac{yz}{2} = \frac{1}{2}.\frac{ax}{2}$$

$$\therefore \quad yz = \tfrac{1}{2}ax \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

But by similar triangles, $\frac{z}{y} = \frac{a}{x}$ $\quad \therefore \quad y = \frac{zx}{a}$ . . (2)

substituting the value of $y$, from (1) we have

$$\frac{z^2x}{a} = \frac{ax}{2} \quad \therefore \quad z = \frac{a}{\sqrt{2}} = \frac{10}{\sqrt{2}} = 7{\cdot}071$$

whence distance from base $= 10 - 7{\cdot}071 = 2{\cdot}929$.

PROB. 69 *a*.

Let $x =$ the less side of the rectangle, and $y =$ the greater side; then $2x =$ greater side of triangle, and $y =$ its hypothenuse;

$\therefore \sqrt{y^2-4x^2} =$ third side of triangle;

also $2x =$ side of square whose area equals that of the two allotments.

$$\therefore 4x^2 = xy + x\sqrt{y^2-4x^2} \ldots\ldots\ldots (1)$$

Length of paling for rectangle and triangle $=$

$$2(x+y) + 2x + y + \sqrt{y^2-4x^2}$$

and length of paling for square $= 8x$

$$\therefore 8x + 55 = 2(x+y) + 2x + y + \sqrt{y^2-4x^2} \ldots (2)$$

From equation (1) we find $y = \frac{5x}{2}$ which substituted in (2) gives $x = 11 \quad \therefore y = 27{\cdot}5$

hence, area rectangle $= xy = 302{\cdot}5$

and area triangle $= x\sqrt{y^2-4x^2} = 181{\cdot}5$.

PROB. 70 *a*.

Let $x+y$, $x$, and $x-y$ represent the three sides of the right-angled triangle;

then $(x+y)^2 = x^2+(x-y)^2 \ldots\ldots\ldots (1)$

and $\frac{1}{2}(x-y).x = 216 \ldots\ldots\ldots\ldots (2)$

From these equations the values of $x$ and $y$ are found, and thence the sides of triangle.

PROB. 71 *a*.

Let $x =$ one side of equilateral triangle;

then area $= \frac{1}{2}x^2 \sin. 60° = \frac{1}{2}x^2.\frac{1}{2}\sqrt{3}$ (Trig. Part II. p. 42.)

$= \frac{x^2}{4}\sqrt{3}$

$$\therefore \text{ expense of paving} = 8 \cdot \frac{x^2}{4} \cdot \sqrt{3} = 2x^2\sqrt{3}$$

and expense of pallisading three sides $= 84 \times 3x = 252x$

$$\therefore 2x^2\sqrt{3} = 252x, \text{ and } x = 72{\cdot}74.$$

PROB. 72 *a* (Fig. Part II. p. 114.)

Let AB (one side of the regular polygon ABDG) $= a$, number of sides $= n$;

$$\therefore \text{ angle AOB} = \frac{1}{n} \cdot 360^\circ$$

$$\text{and angle AOK} = \frac{1}{n} \cdot 180^\circ$$

Now area polygon $= n \times$ area triangle AOB

$= 2n \times$ area triangle AOK

$= 2n \cdot \frac{1}{2}\text{AK} \cdot \text{KO} = n \cdot \text{AK} \cdot \text{KO}$

$$\text{but KO} = \text{AK cot. AOK} = \text{AK cot.} \frac{180^\circ}{n}$$

$$\therefore \text{ area polygon} = n\text{AK}^2 \cdot \text{cot.} \frac{180^\circ}{n}$$

$$= \frac{na^2}{4} \text{ cot.} \frac{180^\circ}{n} \quad \text{since AK} = \frac{a}{2}$$

Ex. If $n = 8$, $a = 16$ yards $\Big\}$ $\therefore$ area $= 512$ cot. $22^\circ\ 30'$

log. 512 . . 2·709270

log. cot. $22^\circ\ 30'$ 0·382776

log. area . . 3·092046 $\therefore$ area $= 1236{\cdot}1$ yards.

PROB. 73 *a*.

In last problem, area $\text{A} = \frac{na^2}{4} \text{ cot.} \frac{180^\circ}{n}$

$$\therefore a^2 = \frac{4\text{A}}{n \text{ cot.} \frac{180^\circ}{n}} = \frac{4\text{A}}{n} \text{tan.} \frac{180^\circ}{n}$$

$$\therefore\ a = \sqrt{\left(\frac{4A}{n}\tan.\frac{180°}{n}\right)}$$

Ex. 1. Let $A = 1236{\cdot}1$, $n = 8$

then $a = \sqrt{\left(\frac{4\times 1236{\cdot}1}{8}\tan.\ 22°30'\right)}$

| | |
|---|---|
| log. 4 . . . . . | 0·602060 |
| log. 1236·1 . . . . | 3·092043 |
| log. tan. $22\frac{1}{2}°$ . . . | 9·617224 |
| ar. co. log. 8 . . . | 9·096910 |
| | 2)2·408237 |
| log. $a$ . . . . . | 1·204118 |

$\therefore\ a = 16.$

PROB. 74 *a* (Fig. 58.)

The area of triangle ABC $= \frac{1}{2}$AB.AC sin. BAC (Part II. p. 53) $= \frac{1}{2}n$AD.AC sin. BAC, (1); since $n$AD = AB.

In triangle AED, $\frac{AE}{AD} = \frac{\text{sin. ADE}}{\text{sin. AED}} = \frac{\text{sin. BAC}}{\text{sin. CAG}}$

$\therefore$ AE sin. CAG = AD sin. BAC.

Substituting this value of AD sin. BAC in (1),

area triangle ABC $= \frac{1}{2}n$AC.AE sin. CAG;

but AC.AE = $AF^2$, since AF is a mean proportional to AC and AE.

$\therefore$ Area triangle ABC $= \frac{1}{2}n.AF^2$ sin. CAG

$= n$ triangle AFH = area polygon.

PROB. 75 (Fig. 60).

Let CB represent length of ship = 160 feet, A the place of the observer; then CAB = 20′ 15″; also the angles SBC, SCE are nearly equal to each other, since the side CB (subtended by an angle of a few minutes) is small compared with AB or AC.

$\therefore$ either of the angles SBC or SCE may be considered to be the course $= 22° 30'$. (Part I. p. 104.)

In triangle ABC, $\frac{AB}{BC} = \frac{\sin. C}{\sin. A} = \frac{\sin. 22° 30'}{\sin. 20' 15''}$

$\therefore$ AB $=$ BC sin. $22° 30'$ cosec. $20' 15'' = 10247$ feet $=$ 1·94 miles.

Otherwise (by circular measure).

Let AD $=$ AB, join DB, then (the angle A being small) each of the angles ABD, ADB is nearly equal to a right angle; and DB may be considered as the arc subtending the angle A. The circular measure of the angle $A = \theta = \frac{\text{arc}}{\text{rad.}} = \frac{DB}{AB} = \frac{20' 15''}{57°·29577}$

$\therefore$ $AB = DB \frac{57°·29577}{20' 15''} = \frac{BC \sin. 22° 30' \times 57°·29577}{20' 15''}$

since BD $=$ BC sin. BCD (considering CDB as a right-angled triangle).

Reducing to seconds, and dividing by $1760 \times 3$ to obtain the distance in miles, we have

$$AB = \frac{160 \times \sin. 22° 30' \times 57·29577 \times 60 \times 60}{1215 \times 1760 \times 3} = 1·94$$

miles.

Prob. 76 *a* (Fig. 53).

Let BC represent the meridian; angle ABC $= x°$, and angle ACB $= x° + 10°$; then A is the point where the ships meet; and the perpendicular AD (the distance from the meridian) $= 100$ miles. Since the sides AB and AC are to each other as 3 : 2, let $AB = 3y$, and $AC = 2y$.

Then $\frac{3y + 2y}{3y - 2y} = \frac{\tan. \frac{1}{2}(x + 10 + x)}{\tan. \frac{1}{2}(x + 10 - x)}$ (Part II. form. C.)

$$\text{or, } 5 = \frac{\tan.(x+5)}{\tan. 5} \quad \therefore \ \tan.(x+5) = 5 \tan. 5°$$

whence $x+5 = 23°37'45''$, and $x = 18°37'45''$
and difference of lat. $BC = BD + DC$

$$= AD \cot. x + AD \cot.(x+10)$$
$$= 296{\cdot}6 + 183{\cdot}2 = 479{\cdot}8.$$

PROB. 77 *a* (Fig. 62).

Let AB represent the height = 2 yards, AA′ the diameter of the earth; draw the tangent BD; then BD will be the distance seen by spectator (neglecting the effects of refraction, &c.)

By Geometry . . . . $BD^2 = BA.BA' = 2 \times 14080002$
(if we suppose diameter = 8000 miles = 14080000 yards)
hence BD = 5307 yards.

PROB. 78 *a* (Fig. 62).

Let BH be drawn perpendicular to BA; then the angle HBD (the angle of depression of D, a point in the horizon) = 90° − CBD = DCB; since angle at D = 90°

$$\therefore \text{ angle DCB} = 2° 13' 27''.$$

Let $a$ = BA the height of the mountain = 3 miles
$x$ = CA or CD the radius of the earth;

$$\text{then sec. C} = \frac{CB}{CD} = \frac{x+a}{x} = 1 + \frac{a}{x}$$

$$\therefore \ \frac{a}{x} = \text{sec. C} - 1 = \frac{1}{\cos. C} - 1 = \frac{1 - \cos. C}{\cos. C} = \frac{2 \sin.^2 \frac{C}{2}}{\cos. C}$$

$$\therefore \ 2x = a \cos. C \operatorname{cosec.}^2 \frac{C}{2}$$

whence, in logarithms,

$$\log. 2x = \log. 3 + \log. \cos. C + 2 \log. \operatorname{cosec}. \frac{C}{2} - 30$$

$\therefore$ diameter $2x = 7952$ miles.

PROB. 79 *a* (Fig. 62).

Let $a =$ DC or AC, the radius of earth

$x =$ BA, height of mountain.

Then $\dfrac{x+a}{a} = \sec. C$

or, $\dfrac{x}{a} = \sec. C - 1 = \dfrac{1 - \cos. C}{\cos. C} = \dfrac{2 \sin.^2 \frac{C}{2}}{\cos. C}$

$\therefore \quad x = 2a \sin.^2 \dfrac{C}{2} \sec. C = 1{\cdot}402$ miles.

PROB. 80 *a* (Fig. 63).

First solution (geometrically).

Upon the line AB = 400 yards, the distance between the objects A and B, describe a segment of a circle containing an angle equal to the given angle (35° 10′) (Euclid III. 33), and measure a perpendicular distance CD = 560 yards, the width of the river; through D draw EDF parallel to base AB, cutting the circle in E and F; then either of the points E, F will correspond to the station on the other side of the river; and the lines EA, EB, or FA, FB, measured on the same scale as AB, will be the required distances.

Second solution.

(1). In right-angled triangle AGC, given AC = 200 and angle AGC = AEB = 35° 10′, to find radius AG = 347·75, and GC = 283·87, and angle CAG = 54° 50′.

(2). $\therefore$ GD = CD − GC = 560 − 283·87 = 276·13.

(3). In right-angled triangle EDG are given GD = 276·13, radius EG = 347·75, to find angle EGD = 37° 19′ 30″.

(4). Then the angle AGE = 180° − (AGC + EGD) = 180° − (35° 10′ + 37° 19′ 30″) = 107° 30′ 30″.

(5). Draw GH perpendicular to AE. We have now given AG and angle AGH = 53° 45′ 15″ in right-angled triangle AGH, to find AH = ½AE, whence AE = 560·1.

(6). To find the other side, EB, there are given AE and AB, and included angle EAB = CAG + GAH = 54° 50′ + 36° 14′ 45″ = 91° 4′ 45″, to find EB = 694·37.

Third solution (analytically).

By the geometrical construction of the fig. (see first solution) we find that the two objects are on the same side of the station. Let A be the nearest object to station, B the more distant.

AB (distance of objects) = $a$ = 400; produce BA to K, meeting perpendicular from station E, and let AK = $x$, AE = $y$, BE = $z$, angle AEK = $\theta$, angle AEB between objects = $\alpha$ = 35° 10′, and perpendicular EK = $b$ = 560.

In triangle AEK . . . $\frac{x}{b} = \tan.\theta$

In triangle BEK . . . $\frac{a+x}{b} = \tan.(\theta+\alpha) = \frac{\tan.\alpha + \tan.\theta}{1 - \tan.\alpha \tan.\theta}$

$$\therefore\ a+x = \frac{b\tan.\alpha + b\tan.\theta}{1-\tan.\alpha\tan.\theta}$$

$$= \frac{b\tan.\alpha + x}{1 - \tan.\alpha\frac{x}{b}} = \frac{b^2\tan.\alpha + bx}{b - x\tan.\alpha}$$

whence (by substituting the above value of tan. $\theta$)

$$x^2 + ax = \frac{ab - b^2\tan.\alpha}{\tan.\alpha}$$

and solving quadratic, $x = -\frac{a}{2} \pm \sqrt{\left\{\frac{a^2}{4} + \frac{ab - b^2 \tan.\alpha}{\tan.\alpha}\right\}}$

To find numerical value of this expression:

First find $b^2 \tan.\alpha$.

| | |
|---|---|
| log. $b$ ..... 2·748188 | log. 3053 ..... 3·484727 |
| log. $b$ ..... 2·748188 | log. cot. $\alpha$ ..... 0·152087 |
| tan. $\alpha$ ..... 9·847913 | 3·636814 |
| 5·344289 | $\therefore \frac{ab - b^2 \tan.\alpha}{\tan.\alpha} = 4333$ |
| $\therefore\ b^2 \tan.\alpha = 220947$ | $\frac{a^2}{4} = 40000$ |
| and $ab = 224000$ | |
| $\therefore\ ab - b^2 \tan.\alpha = 3053$ | $\therefore \frac{a^2}{4} + \frac{ab - b^2 \tan.\alpha}{\tan.\alpha} = 44333$ |

and $\sqrt{44333} = 210{\cdot}5$ $\therefore\ x = -200 \pm 210{\cdot}5 = 10{\cdot}5$ (using the upper sign; the lower sign giving a corresponding point to the right of B). With this value of $x$ we find AE = 560·1, and EB = 694·3.

PROB. 81 *a* (Fig. 64).

Let AB represent the tower, CD the road, C the place of spectator; then angle ACB = 20° (the elevation of tower at C), and ACD = 30° (the angle between line joining the top and road CD). Let BD be the nearest distance of tower from road (= 200 feet); then plane of triangle ABD is perpendicular to road, and therefore ADC is a right angle.

In triangle ABC . . . AB = AC sin. ACB = AC sin. 20°

In triangle ADC . . . AD = AC sin. ACD = AC sin. 30°

$$\therefore \frac{AB}{AD} = \frac{\text{sin. } 20°}{\text{sin. } 30°} \quad \text{but } \frac{AB}{AD} = \text{sin. ADB}$$

$$\therefore \text{sin. ADB} = \frac{\text{sin. } 20°}{\text{sin. } 30°} \quad \text{or ADB} = 43° 9' 15''$$

then in triangle ABD,

$AB = DB \tan. ADB = 200 . \tan. 43° 9' 15'' = 187{\cdot}5$ feet.

Or thus (by Spherical Trigonometry).

The angle at C is a solid angle, formed by the vertical angle ACB, the oblique angle ACD, and horizontal angle BCD which call $\theta$; let $ACB = \alpha$, $ACD = \beta$. To find the horizontal angle $\theta$ we may consider the point C the center of a sphere, and the three angles, $\alpha$, $\beta$, $\theta$, may then be represented by the three sides of a right-angled spherical triangle (Part II. p. 63), of which $\alpha$ and $\theta$ contain the right angle, and $\beta$ is the hypothenuse.

By Rule XV. Part I., $\cos. \beta = \cos. \alpha \cos. \theta$;

whence $\cos. \theta = \cos. \beta \sec. \alpha$,

which determines $\theta = 22° 50' 15''$;

then in right-angled triangle BDC are given BD and angle BCD, to find CB, which with the angle ACB will give AB, the height of tower.

PROB. 82 *a*.

Let $7x$ and $4x$ represent the sides, and included angle $= 129° 34'$; let $\theta$ and $\varphi$ denote the other two angles.

$\therefore \theta + \varphi = 180° - 129° 34' = 50° 26'$.

Now $\dfrac{7x+4x}{7x-4x} = \dfrac{\tan. \frac{1}{2}(\theta+\varphi)}{\tan. \frac{1}{2}(\theta-\varphi)}$ . . . (Trig. Part II. p. 52.)

or $\dfrac{11}{3} = \dfrac{\tan. 25° 13'}{\tan. \frac{1}{2}(\theta-\varphi)}$ $\therefore \tan. \frac{1}{2}(\theta-\varphi) = \dfrac{3}{11} \tan. 25° 13'$

which determines $\theta - \varphi$; and therefore with $\theta + \varphi$ already known, the angles $\theta$ and $\varphi$ may be found.

PROB. 83 *a*.

First solution.

Let $\theta$, $2\theta$, $3\theta$ denote the angles; $x$, $y$, $z$ the sides opposite to them respectively;

then $x+y+z=6$ ..... (1)

$\theta+2\theta+3\theta=180°$ $\therefore\ \theta=30°$

Now $\frac{x}{z}=\frac{\sin.\theta}{\sin.3\theta}=\frac{\sin.30°}{\sin.90°}=\frac{1}{2}$ $\therefore\ 2x=z$.... (2)

and $\frac{x}{y}=\frac{\sin.\theta}{\sin.2\theta}=\frac{\sin.30°}{\sin.60°}=\frac{\frac{1}{2}}{\frac{1}{2}\sqrt{3}}=\frac{1}{\sqrt{3}}$

$\therefore\ x\sqrt{3}=y$ ..... (3)

Substituting in (1) the values of $y$ and $z$ in (3) and (2), we have

$$x+x\sqrt{3}+2x=6, \text{ or } x=\frac{6}{3+\sqrt{3}}=3-\sqrt{3}=1\cdot268$$

whence $z=2x=2\cdot536$, and $y=x\sqrt{3}=2\cdot196$

Second Solution.

This problem, and Numbers 84, 85, and 96, in which are given (or may be found) the perimeter and all the angles, can be solved in the following manner.

Let ABC be the triangle (fig. 20).

To find side $a$.

By (C), (D), Part II. p. 52,

$$bc\sin.^2\frac{A}{2}=\tfrac{1}{2}(a+b+c).\tfrac{1}{2}(a-b+c)$$

$$ac\cos.^2\frac{B}{2}=\tfrac{1}{2}(a+c+b).\tfrac{1}{2}(a+c-b)$$

$$ab\cos.^2\frac{C}{2}=\tfrac{1}{2}(a+b+c).\tfrac{1}{2}(a+b-c)$$

$$\therefore\ \frac{a^2bc\cos.^2\frac{B}{2}\cos.^2\frac{C}{2}}{bc\sin.^2\frac{A}{2}}=\frac{\frac{1}{4}(a+b+c)^2.\frac{1}{2}(a+c-b).\frac{1}{2}(a+b-c)}{\frac{1}{2}(a+b-c).\frac{1}{2}(a-b+c)}$$

$$\therefore\ a=\tfrac{1}{2}(a+b+c).\sin.\frac{A}{2}\sec.\frac{B}{2}\sec.\frac{C}{2}$$

*Calculation.*

Let $A = \theta$, $B = 2\theta$, $C = 3\theta$.

$\therefore\ 180° = 6\theta\ \therefore\ \theta = 30° = A$, $B = 60°$, and $C = 90°$, and $a+b+c = 6$.

| | |
|---|---|
| log. 3 | 0·477121 |
| log. sin. $\frac{1}{2}$A | 9·412996 |
| log. sec. $\frac{1}{2}$B | 0·062469 |
| log. sec. $\frac{1}{2}$C | 0·150515 |
| log. $a$ | 0·103101 |

$$\therefore\ a = 1{\cdot}268$$

PROB. 84 *a*.

Let $\theta$, $2\theta$, $4\theta$ denote the angles; $x$, $y$, $z$ the sides opposite to them respectively;

then $x+y+z = 100$ ..... (1)

and $\theta+2\theta+4\theta = 180°$

$$\therefore\ \theta = 25°\,42'\,51''.$$

Now $\dfrac{z}{y} = \dfrac{\sin. 4\theta}{\sin. 2\theta} = \dfrac{2 \sin. 2\theta \cos. 2\theta}{\sin. 2\theta} = 2 \cos. 2\theta = 1{\cdot}247$

$$\therefore\ z = 1{\cdot}247y$$

$$\frac{y}{x} = \frac{\sin. 2\theta}{\sin. \theta} = \frac{2 \sin. \theta \cos. \theta}{\sin. \theta} = 2 \cos. \theta = 1{\cdot}802$$

$$\therefore\ x = \frac{y}{1{\cdot}802}$$

substituting in (1),

$$\frac{y}{1{\cdot}802} + y + 1{\cdot}247y = 100$$

$$\therefore\ y = 35{\cdot}69$$

whence $x = 19{\cdot}8$, and $z = 44{\cdot}51$.

Otherwise, see Prob. 83, second solution.

PROB. 85 *a*.

Let $x$, $y$, $z$ denote the base, perpendicular, and hypothenuse respectively;

then $x+y+z=24$ ..... (1)

$x=z\cos.30°=\frac{1}{2}\sqrt{3}.z$ ..... (2)

$y=z\sin.30°=\frac{1}{2}z$ ..... (3)

$\therefore (\frac{1}{2}\sqrt{3}+\frac{1}{2}+1)z=24$

$\therefore z=10{\cdot}144$

whence $x=8{\cdot}784$, $y=5{\cdot}072$.

Otherwise, see Prob. 83, second solution.

PROB. 86 *a* (Fig. 20).

Let the two sides AC and CB be denoted by $x$ and $y$; then we have given $x+y=600$, the side $a=400$, and the angle $A=80°$, to solve the triangle.

By (D) Part II. p. 52,

$$xy\cos.^2\frac{A}{2}=\tfrac{1}{2}(x+y+a).\tfrac{1}{2}(x+y-a)$$

$$xy\cos.^2 40°=\tfrac{1}{2}(600+400).\tfrac{1}{2}(600-400)$$

$$=500.100$$

$$\therefore xy=50000.\sec.^2 40°$$

or $xy=85204$.

Hence, knowing the values of $x+y$ and $xy$, the sides $x$ and $y$ may be found.

PROB. 87 *a*.

By (C) Part II. p. 52,

$$bc\sin.^2\frac{A}{2}=\tfrac{1}{2}(a+b-c).\tfrac{1}{2}(a-b+c).$$

where $a+b=600$, $c=400$, and $A=60°$

and $\therefore \sin.^2\frac{A}{2}=\frac{1}{4}$

$$\therefore \quad 400b \cdot \tfrac{1}{4} = \tfrac{1}{2}(200) \cdot \tfrac{1}{2}(a-b+400)$$
$$\text{or } 400b = 200 \cdot (a-b) + 80000$$
$$\text{and } 2b = a-b+400$$
$$\therefore \quad 3b-400 = a = 600-b$$
$$\therefore \quad 4b = 1000$$
$$\text{and } b = 250$$
$$\text{whence } a = 350$$

PROB. 88 *a*.

Let $z$ represent the hypothenuse and $x$ the perpendicular;

$$\text{then } z+x+8=24 \ldots\ldots\ldots (1)$$
$$\text{and } z^2=x^2+64 \ldots\ldots\ldots (2)$$
$$\text{whence } z=10, \quad x=6.$$

PROB. 89 *a* (Fig. 21).

Let height of steeple AB = $x$, distance CB of first station from steeple = 80, distance DB of second station = 250.

Let angle ACB = $2\alpha$, then angle ADB = $\alpha$;
and since angle ACB = angle D + angle DAC

$$\therefore \quad 2\alpha = \alpha + \text{angle DAC}$$

$\therefore$ angle DAC = $\alpha$; or triangle ADC is isosceles, and AC = DC = 170;

$$\text{hence } x = \sqrt{AC^2-CB^2} = \sqrt{170^2-80^2}$$
$$= \sqrt{(170+80)\cdot(170-80)}$$
$$= \sqrt{250\times 90} = \sqrt{25\times 900} = 5\times 30 = 150.$$

PROB. 90 *a*.

Let the sides AB, AC of a triangle ABC be denoted by $x$ and $y$; then we have given angle A, side $a$, and $x+y=b$, to solve the triangle.

By (D) Part II. p. 52,

$$xy\cos.^2\frac{A}{2}=\tfrac{1}{2}(x+y+a).\tfrac{1}{2}(x+y-a)$$
$$=\tfrac{1}{4}(b+a).(b-a)$$
$$\therefore\quad xy=\tfrac{1}{4}(b+a).(b-a)\sec.^2\frac{A}{2}$$

From this equation, and $x+y=b$, the values of $x$ and $y$ may be found.

### Prob. 91 *a*.

Let the sides AB, AC of a triangle ABC be denoted by $x$ and $y$; then we have given the angle A, side $a$, and $x-y=d$, to solve the triangle.

By (C) Part II.

$$xy\sin.^2\frac{A}{2}=\tfrac{1}{2}(a+\overline{x-y}).\tfrac{1}{2}(a-\overline{x-y})$$
$$=\tfrac{1}{4}(a+d).(a-d)$$
$$\therefore\quad xy=\tfrac{1}{4}(a+d).(a-d).\mathrm{cosec.}^2\frac{A}{2}$$

From this equation, and $x-y=d$, the values of $x$ and $y$ may be found.

### Prob. 92 *a*.

Let A denote the angle opposite base $a$, X and Y the angles at the base, and $x$ and $y$ the sides opposite to X and Y;

then $x+y=b$ ..... (1)

and $X-Y=d$.

Now $\dfrac{\text{sin. X}}{\text{sin. A}}=\dfrac{x}{a}$ and $\dfrac{\text{sin. Y}}{\text{sin. A}}=\dfrac{y}{a}$

adding, $\dfrac{\text{sin. X}+\text{sin. Y}}{\text{sin. A}}=\dfrac{x+y}{a}=\dfrac{b}{a}$

$$\text{or} \quad \frac{\sin. X+\sin. Y}{\sin. (X+Y)} = \frac{b}{a} \quad \text{since } A=180-\overline{X+Y}$$

$$\text{or} \quad \frac{2\sin.\tfrac{1}{2}(X+Y)\cos.\tfrac{1}{2}(X-Y)}{2\sin.\tfrac{1}{2}(X+Y)\cos.\tfrac{1}{2}(X+Y)} = \frac{b}{a}$$

$$\text{or} \quad \frac{\cos.\tfrac{1}{2}d}{\cos.\tfrac{1}{2}(X+Y)} = \frac{b}{a}$$

$$\therefore \quad \cos.\tfrac{1}{2}(X+Y) = \frac{a}{b}\cos.\tfrac{1}{2}d.$$

PROB. 93 *a*.

Let $a$ be the base and $x$, $y$ the two sides of triangle; X, Y the angles opposite to $x$, $y$ respectively, and A the angle opposite base $a$.

Then $x-y=d$, and $X-Y=D$.

$$\frac{\sin. X}{\sin. A} = \frac{x}{a} \qquad \frac{\sin. Y}{\sin. A} = \frac{y}{a}$$

$$\therefore \text{ subtracting, } \frac{\sin. X-\sin. Y}{\sin. A} = \frac{x-y}{a} = \frac{d}{a}$$

$$\text{or} \quad \frac{2\cos.\tfrac{1}{2}(X+Y)\sin.\tfrac{1}{2}(X-Y)}{\sin. (X+Y)} = \frac{d}{a}$$

$$\text{or} \quad \frac{2\cos.\tfrac{1}{2}(X+Y.)\sin.\tfrac{1}{2}(X-Y)}{2\sin.\tfrac{1}{2}(X+Y)\cos.\tfrac{1}{2}(X+Y)} = \frac{d}{a}$$

$$\text{whence } \sin.\tfrac{1}{2}(X+Y) = \frac{a}{d}\sin.\tfrac{1}{2}(X-Y)$$

$$= \frac{a}{d}\sin.\tfrac{1}{2}D$$

and since $X-Y$ is also known, the sides and angles of triangle may be found.

PROB. 94 *a*.

Let ABC (fig. 20) be any triangle; then angle A, side $b$, and the difference ($a - c = d$) between $a$ and $c$ are given, to solve the triangle.

By (C) Part II.

$$bc \sin.^2 \frac{A}{2} = \tfrac{1}{2}(a+b-c).\tfrac{1}{2}(a-b+c)$$

By (D)... $bc \cos.^2 \frac{A}{2} = \tfrac{1}{2}(b+c+a).\tfrac{1}{2}(b+c-a)$

$$\therefore \quad \tan.^2 \frac{A}{2} = \frac{(a+b-c).(a+c-b)}{(b+c+a).(b+c-a)}$$

similarly, $\tan.^2 \frac{C}{2} = \frac{(c+b-a).(c+a-b)}{(b+a+c).(b+a-c)}$

$$\therefore \quad \frac{\tan.^2 \frac{C}{2}}{\tan.^2 \frac{A}{2}} = \left(\frac{b+c-a}{a+b-c}\right)^2$$

$$\therefore \quad \tan. \frac{C}{2} = \frac{b+c-a}{a+b-c}.\tan. \frac{A}{2} = \frac{b-(a-c)}{b+(a-c)}.\tan. \frac{A}{2}$$

$$= \frac{b-d}{b+d}.\tan. \tfrac{1}{2}A$$

PROB. 95 *a*.

Let ABC (fig. 20) be any triangle; then angle A, side $b$, and the sum $(a+c=m)$ of $a$ and $c$ are given, to solve the triangle.

As in last problem,

$$\tan.^2 \frac{A}{2} = \frac{(a+b-c).(a+c-b)}{(b+c+a).(b+c-a)}$$

$$\tan.^2 \frac{C}{2} = \frac{(c+b-a).(c+a-b)}{(b+a+c).(b+a-c)}$$

$$\therefore \quad \tan. \frac{A}{2} \tan. \frac{C}{2} = \frac{a+c-b}{a+b+c} = \frac{m-b}{m+b}$$

$$\therefore \quad \tan. \frac{C}{2} = \frac{m-b}{m+b}.\cot. \frac{A}{2}$$

PROB. 96.

See Problem 83, second solution.

PROB. 97 *a*.

In fig. 21, produce BCD to a point E, and join AE. Let AB represent the object, and C, D, E the three stations.

DC=*a*, ED=*b*, the angle ACB=2$\theta$, ADC=90°−$\theta$, and AED=$\theta$.

The triangle AEC is manifestly isosceles, since angle ACB=AEC+EAC, or 2$\theta$=$\theta$+EAC, $\therefore$ EAC=$\theta$, or AC=EC=*a*+*b*.

In triangle AEC ..... $\frac{AE}{a+b} = \frac{\sin. 2\theta}{\sin. \theta}$

$$\therefore \quad AE = (a+b).\frac{\sin. 2\theta}{\sin. \theta}$$

In triangle AED .... $\frac{AE}{b} = \frac{\sin. (90°-\theta)}{\sin. (90°-2\theta)} = \frac{\cos. \theta}{\cos. 2\theta}$

$$\therefore \quad AE = \frac{b \cos. \theta}{\cos. 2\theta}$$

$$\therefore \quad \frac{b \cos. \theta}{\cos. 2\theta} = \frac{(a+b) \sin. 2\theta}{\sin. \theta}$$

$$\text{or} \quad \cos. 2\theta = \frac{b}{2(a+b)}$$

In triangle ACB .... CB = $(a+b) \cos. 2\theta$

$$\therefore \quad CB = \tfrac{1}{2}b$$

hence $AB^2 = AC^2 - CB^2 = (a+b)^2 - (\tfrac{1}{2}b)^2$

$\therefore$ height $AB = \sqrt{(a+b+\tfrac{1}{2}b).(a+b-\tfrac{1}{2}b)}$

$$= \sqrt{(a+\tfrac{3}{2}b).(a+\tfrac{1}{2}b)}$$

if $a=b=20$ $\quad = \sqrt{50 \times 30} = 38{\cdot}73$

PROB. 98 (Fig. 38).

Let the angles BDC, ADC be denoted by $\alpha$ and $\beta$

respectively, and the sides of the triangle ABC by $a$, $b$, $c$, in the usual manner. Let angle CAD$=x$, and CBD$=y$; then in triangles BDC and ADC, we have

$$\frac{a \sin. y}{\sin. \alpha} = \text{CD} = \frac{b \sin. x}{\sin. \beta}$$

$$\therefore \quad \frac{\sin. x}{\sin. y} = \frac{a \sin. \beta}{b \sin. \alpha}$$

or
$$\frac{\sin. x + \sin. y}{\sin. x - \sin. y} = \frac{a \sin. \beta + b \sin. \alpha}{a \sin. \beta - b \sin. \alpha}$$

(Part II. p. 35.)
$$\frac{\tan. \frac{1}{2}(x+y)}{\tan. \frac{1}{2}(x-y)} = \frac{a \sin. \beta + b \sin. \alpha}{a \sin. \beta - b \sin. \alpha}$$

whence
$$\tan. \tfrac{1}{2}(x-y) = \frac{1 - \dfrac{b \sin. \alpha}{a \sin. \beta}}{1 + \dfrac{b \sin. \alpha}{a \sin. \beta}} . \tan. \tfrac{1}{2}(x+y)$$

and $x+y$, the sum of the angles, is known, since it is $= 360° - \text{C} - \alpha - \beta$, and by the equation $x-y$ is given, hence the angles $x$ and $y$ may be found.

Applying this expression to an example, suppose that $\alpha = 54° 13' 45''$, $\beta = 33° 52'$, $\text{C} = 77° 42' 24''$, and that $\log. a = 4{\cdot}170262$, $\log. b = 4{\cdot}021189$.

First find the value of $\dfrac{b \sin. \alpha}{a \sin. \beta}$, which is $1{\cdot}0329$,

$$\text{then } \tan. \tfrac{1}{2}(x-y) = \frac{-0{\cdot}0329}{2{\cdot}0329} \tan. \tfrac{1}{2}(x+y)$$

$$\text{or } \overset{+}{\tan.} \tfrac{1}{2}(x-y) = \frac{-329}{20329} \overset{-}{\tan.} 97° 5' 55''$$

(placing the proper sign over tan. $97° 5' 55''$, and thus determining the sign of tan. $\frac{1}{2}(x-y)$, (see Rule 14, Part I.)

$$\begin{aligned} \text{whence } \tfrac{1}{2}(x-y) &= 7^\circ\ 24'\ 24'' \\ \text{and } \tfrac{1}{2}(x+y) &= 97\ \ 5\ \ 55 \\ \therefore\ x &= 104\ \ 30\ \ 19 \\ y &= 89\ \ 41\ \ 31 \end{aligned}$$

All the angles of the triangles ACD and CDB being now known, it will be easy to calculate the distances AD, CD, BD. If we perform the operation, we shall find AD = 12516, CD = 18241, and AD = 10742·5.

Otherwise:

This problem is more easily solved by using a subsidiary angle; thus,

$$\text{assume } \tan.\theta = \frac{b \sin.\alpha}{a \sin.\beta} \quad \ldots\ldots\ldots\ldots (1)$$

$$\text{then } \tan.\tfrac{1}{2}(x-y) = \frac{1-\tan.\theta}{1+\tan.\theta} \cdot \tan.\tfrac{1}{2}(x+y)$$
$$= \tan.(45^\circ-\theta) \cdot \tan.\tfrac{1}{2}(x+y)$$

Find $\theta$ from equation (1), which is 45° 55′ 45″, whence $\frac{1}{2}(x-y) = 7^\circ\ 25'$, and the values of $x$ and $y$ may then be found as before.

PROB. 99 *a*.

Let straight lines OA, OB, OC, drawn at right angles to one another, form at the point O the solid angle of a cube.

Let OA = $a$, OB = $b$, OC = $c$; join the extremities A, B, C, and let the sides of the triangle ABC be denoted by $a_1$, $b_1$, $c_1$;

$$\text{then area ABC} = \tfrac{1}{2} b_1 c_1 \sin.A = \tfrac{1}{2} b_1 c_1 \sqrt{1-\cos.^2 A}$$
$$= \tfrac{1}{2} b_1 c_1 \sqrt{1-\left(\frac{b_1^2+c_1^2-a_1^2}{2b_1c_1}\right)^2}$$
$$= \tfrac{1}{4}\sqrt{4b_1^2c_1^2-(b_1^2+c_1^2-a_1^2)^2}$$

Now in right-angled triangles AOC, AOB, and BOC,

we have
$$b_1^2 = a^2 + c^2$$
$$c_1^2 = a^2 + b^2$$
$$a_1^2 = b^2 + c^2$$
$$\therefore\ b_1^2 + c_1^2 - a_1^2 = 2a^2$$
and
$$4b_1^2 c_1^2 = 4(a^2+c^2).(a^2+b^2)$$
$$= 4(a^4 + a^2b^2 + a^2c^2 + b^2c^2)$$

substituting these values, we have

$$\text{area ABC} = \tfrac{1}{2}\sqrt{a^2b^2 + a^2c^2 + b^2c^2}.$$

PROB. 100 *a*.

Let ABC be an equilateral triangle, and upon the sides describe squares, the centers of which are A′, B′, and C′, and join the points A′, B′, C′, forming the equilateral triangle A′B′C′, A′ being opposite A, &c., then,

$$\text{area A′B′C′} = \text{area ABC} \times (1 + \tfrac{1}{2}\sqrt{3}).$$

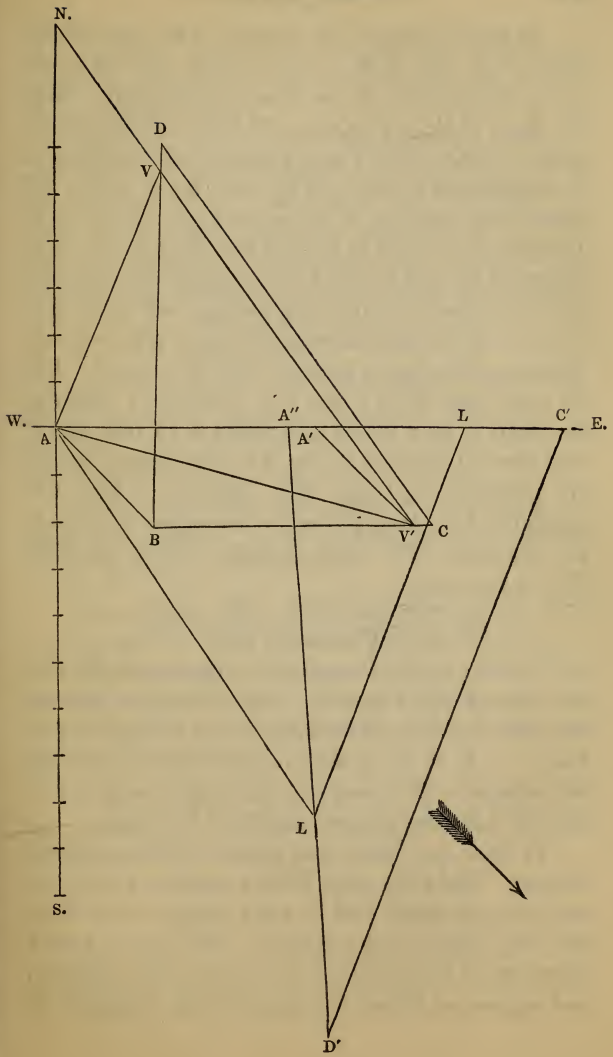
For
$$\text{area A′B′C′} = \tfrac{1}{2}\text{B′C′}^2.\ \sin.\ 60°$$
$$\text{area ABC} = \tfrac{1}{2}\text{AC}^2.\ \sin.\ 60°$$
$$\therefore\ \text{area A′B′C′} = \frac{\text{B′C′}^2}{\text{AC}^2}.\ \text{area ABC}.$$

Now to find the value of B′C′ in terms of AC.

In triangle AB′C′,
$$2\text{AB′}.\ \text{AC′}\cos.\ \text{B′AC′} = \text{AB′}^2 + \text{AC′}^2 - \text{B′C′}^2$$
or
$$2\text{AB′}^2 \cos.\ 150° = 2\text{AB′}^2 - \text{B′C′}^2$$
$$\therefore\ \text{B′C′}^2 = 2\text{AB′}^2(1 - \cos.\ 150°)$$
$$= 2\text{AB′}^2(1 + \cos.\ 30°)$$
$$= 2\text{AB′}^2(1 + \tfrac{1}{2}\sqrt{3})$$
$$\text{But AB′}^2 = 2.\left(\frac{\text{AC}}{2}\right)^2 = \tfrac{1}{2}\text{AC}^2$$
$$\therefore\ \text{B′C′}^2 = \text{AC}^2(1 + \tfrac{1}{2}\sqrt{3})$$
hence
$$\text{area A′B′C′} = (1 + \tfrac{1}{2}\sqrt{3}).\ \text{area ABC}.$$

PROB. 101 *a*.

The geometrical construction of this problem may be made as follows:


N.
D
V
W.
A''
L
C'
E.
A
A'
B
V'
C
L
S.
D'

To find the course and distance of the vice-admiral. Draw N.S. and W.E. at right angles, and let A, the point of intersection, be the place of the admiral. Make the angle NAV = 2 points, or N.N.E., and take from a scale AV = 6; then V is the place of the vice-admiral. Make SAB = 4 points, or S.E., and AB = 3, then it will simplify the problem if we suppose the admiral to be transferred to B, and to sail due east along the line BV′, instead of along AE, which is the actual line described by the admiral; for then the problem will be reduced to this: to find the course and distance of the vice-admiral, so as to form a junction with the admiral. Join BV, and take BC = 6, the rate of admiral's steaming, and with C as a center and radius = 10 (the rate of vice-admiral) describe an arc of a circle cutting BV (or BV produced) in D; join DC. Through V draw VV′ parallel to DC; then VV′ is the vice-admiral's distance, and the angle ANV which it makes with the meridian N.S. is the course.

$$\text{For } \frac{\text{rate of admiral}}{\text{rate of vice-ad.}} = \frac{BC}{DC} = \frac{BV'}{VV'}$$

∴ V′ is the point of junction, on the hypothesis that the admiral sailed from B. But the admiral steamed along the line AE, and described AA′ = BV′ in the same time; ∴ A′ is the position of the admiral when the vice-admiral is at V′, and it is evident the position of V′ from A′ is the one required, namely, S.E. 3 miles.

To find the course and distance of the look-out steamer. Make the angle SAL = 3 points or S.E.b.S., and AL = 10 miles, then L is the position of the look-out with respect to the admiral. Take AA″ = 5 miles (the distance the steamer is to be a-head of the admiral), and suppose, as before, the admiral to be transferred to

A″; join LA″; then the problem may be solved by supposing the steamer to form a junction with the admiral, as in the former case.

From A″E take A″C′ = 6 miles, the rate of admiral, and with C′ as center, and distance 14 miles, describe an arc cutting A″D′ in D′, and join D′C′. Through L, the place of the steamer, draw LL′ parallel to D′C′. Then LL′ is the steamer's distance run to get into her station, and the angle it makes with the meridian is her course.

$$\text{For } \frac{\text{rate of admiral}}{\text{rate of steamer}} = \frac{A''C'}{C'D'} = \frac{A''L'}{L'L}$$

∴ L′ is the point of junction of the admiral and steamer; but since we have supposed the admiral transferred to a point A″, when in fact he was at A, the position of admiral will be 5 miles to the west of steamer.

To find the course and distance of reinforcement. Join AV′; then AV′ is the distance run, and the angle V′AS the course of reinforcement.

This problem may be solved trigonometrically, thus:

To find the course and distance of vice-admiral.

(1.) In triangle AVB, given AV = 6, AB = 3, and included angle VAB = 10 points = 112° 30′, to find the other angles; hence angle ABV = 46° 18′ 30″, and AVB = 21° 11′ 30″.

(2.) In the triangle BDC are given BC = 6, DC = 10, and the angle DBC = 180° − 45° − 46° 18′ 30″ = 88° 41′ 30″, to find angle BDC = BVV′ = 36° 51′ 30″

adding AVB = 21 11 30

∴ angle AVV′ = 58 3 0 = 5 points nearly.

∴ the course of the vice-admiral = 5 points to the left of admiral's bearing, or 5 points to the left of S.S.W.

∴ course = S.E.b.S. nearly.

(3.) In triangle AVB are given AV = 6, VAB = 10 points or 112° 30′, and angle ABV = 46° 18′ 30″, to find BV = 7·67 miles.

(4.) In triangle VBV′ are given BV = 7·67, angle VBV′ = 88° 41′ 30″, and VV′B = 54° 27′, to find VV′ = 9·42, the distance run by vice-admiral.

To find the course and distance of look-out steamer.

(1.) In triangle AA″L, given AL = 10, AA″ = 5, and included angle A″AL = 5 points or 56° 15′, to find the other angles; hence angle AA″L = 93° 49′ 30″, and ALA″ = 29° 55′ 30″.

(2.) In triangle A″D′C′ are given A″C′ = 6, C′D′ = 14, and angle D′A″C′ = 180° − 93° 49′ 30″ = 86° 10′ 30″, to find angle A″D′C′ = A″LL′ = 25° 19′ 0″

adding ALA″ = 29 55 30

∴ course of steamer = 55 14 30 to the right of ad.
bearing of adm. N.W.b.N. = 33 45 0 left of north

∴ course of look-out = 21 29 30 right of north
or N.N.E. nearly.

(3.) In triangle ALA″ are given AL = 10, angle AA″L = 93° 49′ 30″, and angle A″AL = 5 points = 56° 15′, to find the side LA″ = 8·33.

(4.) In triangle A″LL′ are given LA″ = 8·33, angle LA″L′ = 86° 10′ 30″, and A″L′L = 68° 30′ 30″, to find LL′ = 8·94, the distance run by look-out.

To find the course and distance run of the reinforcement.

(1.) To find BV′ or AA′ the distance run by admiral.

$$BV' : VV'(=9{\cdot}42) :: 6 : 10$$

$$\therefore \quad BV' = 5{\cdot}62$$

Or thus: in triangle VBV′ are given all the angles and side VV′ to find BV′.

(2.) To find distance AV'.

In triangle ABV' are given two sides, AB and BV', and included angle ABV', to find AV' = 8 miles.

(3.) To find course of reinforcement.

In same triangle find angle BAV' = 29° 44' left of B.

bearing of B, S.E. = 45 0 left of S.

bearing of reinforcement = 74 44 left of S.

hence course of reinforcement = E.b.S.$\frac{1}{2}$S. nearly.

To find time of reaching their appointed stations.

10 : 9·42 :: 1 h. : $x$ = 56 min. for vice-admiral

14 : 8·92 :: 1 h. : $x$ = 38 min. for look-out.

To find the rate of steaming of reinforcement, we have

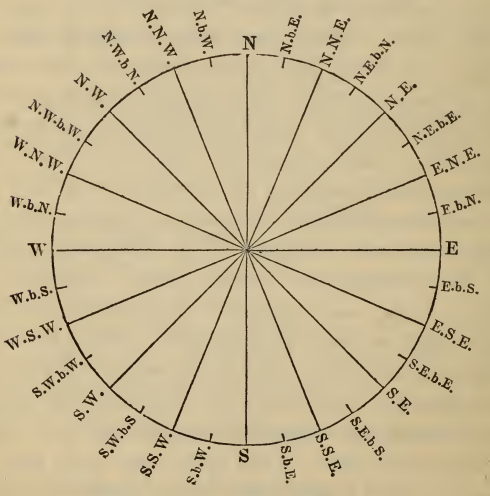
56 m. : 1 h. :: 8 : $z$ = 8$\frac{1}{2}$ miles.

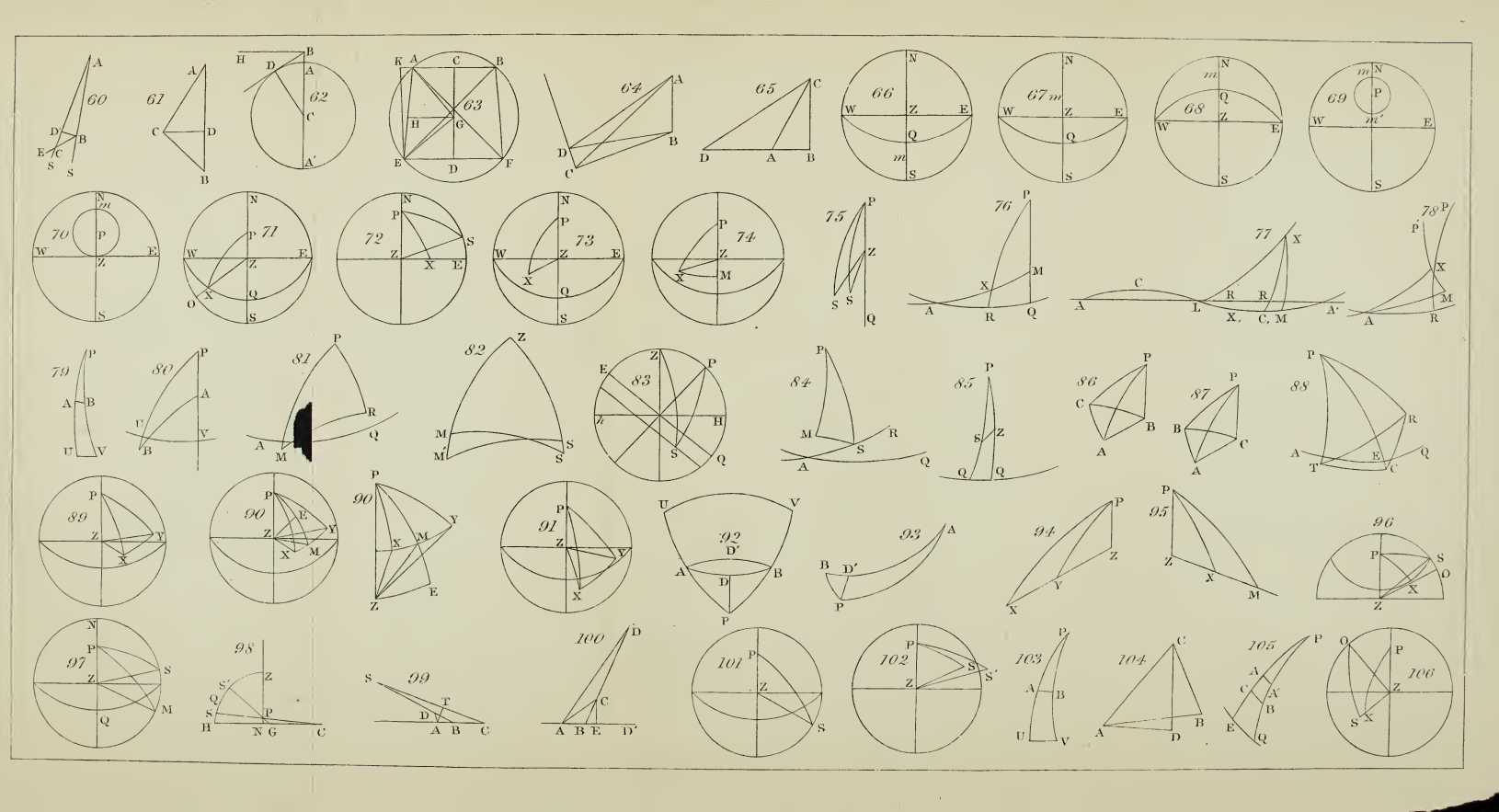
The above belongs to a very important class of problems connected with naval tactics and the evolutions of steam squadrons. A knowledge of these subjects will become of more importance hereafter, when it is found that a fleet of steamers may be handled with almost the same precision as an army; even now our steamers are sufficiently numerous to compose fleets for warlike purposes. Captain C. R. Moorsom's treatise "On the Principles of Naval Tactics" deserves the attention of the naval student. In it he will find problems similar to the one given above, solved by means of tables, by inspection. In fact, to render such problems of any practical value, tables for facilitating the computations must be constructed; but to do this (or even to apply them with ease to any given case when they are formed) requires a knowledge of the principles such as the above problem teaches. The following, taken from Captain Moorsom's book, are here given as exercises to the student in constructing the figure and working out the answer by trigonometry.

(1.) An admiral steering W. five knots, wind south, with a frigate stationed two miles on his larboard quarter bearing S.E., makes signal to her to look out N. twelve miles: how must she steer? *Ans.* N.W.

(2.) Let the admiral steer W. five knots, and the frigate be S.W. (on his larboard bow) twelve miles distant, and her signal be made to look out north, nine miles: how must she steer? *Ans.* N.b.W.$\frac{1}{2}$W.

THE COMPASS.





# SOLUTIONS

OF

# PROBLEMS IN NAVIGATION.

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(1.) Two distinct methods are used for navigating a ship from one place to another; the first is an application of the common rules of plane trigonometry; the other requires a knowledge of spherical trigonometry, and of the principal definitions and facts in astronomy. The latter is for this reason called *Nautical Astronomy;* the characteristic name of the former being *Navigation* or *plane sailing.*

(2.) The necessary angles and measurements in the first method are supplied by means of the compass and log-line; in the second and more exact method they are obtained by astronomical observations.

(3.) The brief investigations of the principal rules in navigation which we intend to give may properly follow in this place, immediately after the problems in surveying, &c., which may be looked upon as introductory to the subject; whilst the astronomical problems will form an appropriate introduction to the rules and investigations in nautical astronomy.

(4.) In pages 29, 30 are given definitions of the principal terms used in navigation; these are, the course, distance, true difference of latitude, difference of longi-

tude, departure, and meridional difference of latitude. The formulæ from which the rules in navigation are derived are the following:

departure = distance . sin. course . . . . . [1]

true diff. lat. = distance . cos. course . . . . . [2]

diff. long. = meridional diff. lat. . tan. course . [3]

In parallel sailing (or when the ship is sailing due east or due west):

distance = diff. long. . cos. lat. . . . . [4]

(5.) We will proceed to prove these four formulæ, and deduce from them the common rules of navigation.

[1]. To prove that departure = dist. . sin. course.

Suppose the ship to sail from A to F (fig. p. 29), cutting all the intermediate meridians at the same angle, PAB, PBC, &c.; this common angle is the course; the arc AF is the distance; and if, in the figure, the triangles ABH, BCI, &c. are taken so small that they may be considered as right-angled plane triangles (p. 30), then HB+IC . . . . +MF is called the departure.

In tri. ABH, HB = AB sin. HAB = AB sin. course

. . . BCI, IC = BC sin. IBC = BC sin. course

. . . . . .

. . . EFM, MF = EF sin. MEF = EF sin. course

$\therefore$ HB+IC . . . +MF=(AB+BC . . . +EF) . sin. course

or departure = distance . sin. course.

[2]. To prove that true diff. lat. = dist. . cos. course.

In the same triangles ABH, BCI, &c.

AH = AB cos. course

BI = BC cos. course

. . . . . .

EM = EF cos. course

$\therefore$ AH+BI . . . +EM=(AB+BC . . . +EF) . cos. course

or true diff. lat. = distance . cos. course.

[3]. To prove that diff. long. = mer. diff. lat. . tan. course.

In a Mercator's chart the meridians are drawn parallel to each other, and perpendicular to the equator, and therefore the parts of parallels of latitude, as BH, CI, &c. are increased and become equal to one another, and to the corresponding parts UV, VW, &c. of the equator (p. 30). To preserve the similarity of the triangular spaces on the chart that correspond to the triangles ABH, &c. the sides AH, AB must be increased in the same proportion that HB has been increased. Hence, when HB+IC . . . . +MF increase and become the difference of longitude, the arcs AH+BI . . . +EM, or AO, will increase and become what is called the meridional difference of latitude between A and F; and, moreover, each of the lines into which AB, BC . . . . EF are projected will be inclined to the parallels at the same angle as the course, since the triangles on the chart are similar to those on the globe; and as they cut the parallels at the same angle, they must be in one and the same straight line. Hence the meridional difference of latitude, difference of longitude, and the line into which the distance AF is projected, form a right-angled plane triangle, in which

$$\frac{\text{diff. long.}}{\text{mer. diff. lat.}} = \text{tan. course}$$

or diff. long. = mer. diff. lat. . tan. course.

### *Parallel Sailing.*

[4]. To prove that distance = diff. long. . cos. lat.

When the course = 90°, or the ship is sailing on a parallel, tan. course = $\infty$; and for this, formula [3] gives no solution. In this case the course is due east or west;

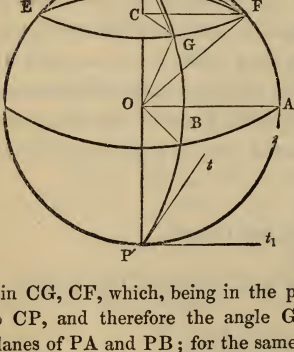
the distance is the arc of the parallel of latitude intercepted between the meridians passing through the two places; and the relation between the distance, diff. long., and latitude, required above, may be found by means of the formula in Trig. Part II. p. 85, namely,

$$\frac{FG}{AB} = \text{cos. } AF^*$$

in which FG is the distance, AB the diff. long., and AF the latitude of the ship. Hence we have this formula for solving all the common problems in parallel sailing:

$$\frac{\text{dist.}}{\text{diff. long.}} = \text{cos. lat.}$$

or dist. = diff. long. . cos. lat.




* This formula is proved in Part II. as follows:

Let FG be the arc of a small circle which is intercepted between the secondaries PA, PB, and which has the same pole, P, as the great circle ABD; then

$$\frac{FG}{AB} = \text{cos. } AF$$

Let the diameter OP cut the plane of GEF in the point C, then C is the center of GEF; join CG, CF, which, being in the plane GEF, are perpendicular to CP, and therefore the angle GCF is the inclination of the planes of PA and PB; for the same reason, the angle AOB is the inclination of the same two planes, therefore the angle GCF = angle AOB. But arcs of circles which subtend equal angles at their centers are proportional to their radii; consequently

$$\frac{FG}{AB} = \frac{CF}{OA} = \frac{CF}{FO} = \text{sin. } POF = \text{cos. } FOA, \text{ or cos. } AF.$$

*Middle Latitude Sailing.*

(6.) Another formula, giving results sufficiently correct when the distance run by the ship is not great (such as in an ordinary day's sailing), is obtained by considering the parallel TS (fig. p. 29) drawn through the middle of AO (the difference of latitude of A and F) as equal to BH+CI . . . . +MF (the departure);

$$\text{for then we have } \frac{\text{TS}}{\text{UZ}} = \text{cos. TU}$$

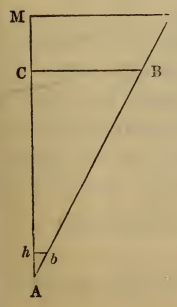
in which TS = departure nearly, UZ = difference of longitude, and TU is the latitude of the middle point between A and F, and therefore called the middle latitude.

$$\text{Hence } \frac{\text{dep.}}{\text{diff. long.}} = \text{cos. mid. lat.}$$

or dep. = diff. long. . cos. mid. lat. . . . . [5]

(7.) The quantities, distance, true diff. lat., departure, and course, may be represented by the three sides and an angle of a right-angled plane triangle. This may be proved as follows:

In fig. p. 29, suppose the distance AF to be divided into $n$ equal parts, AB, BC . . . EF; then AF = $n$ AB; ∴ the true diff. lat. AO (= AH+BI . . .+EM) = $n$ AH,




since the triangles ABH, BCI, &c. are equal in every respect. Make a plane triangle A$bh$ equal and similar to triangle ABH, and produce A$b$ to B and A$h$ to C, so that AB = distance AF, and AC = true diff. lat. AO; join BC; then BC shall be at right angles to AC, and equal to BH+CI . . . . +MF (the departure).

For since $AB = n \cdot Ab$

and $AC = n \cdot Ah$

$\therefore$ AB and AC are cut proportionally in the points $b$ and $h$; hence BC is parallel to $bh$, and therefore at right angles to AC.

Again, in right-angled triangle ABC,

$$BC = AB \text{ sin. } A$$

or $BC =$ dist. sin. course

but, form. [1] p. 122, departure = dist. sin. course

$\therefore$ BC = departure.

(8.) The two quantities, meridional difference of latitude, and difference of longitude, may be represented by two sides of a triangle which is similar to the triangle ABC in last Art.

For let AM (p. 125) be taken equal to the meridional difference of latitude between A and F (fig. p. 29), and the right-angled plane triangle AMN completed; then MN will equal the difference of longitude UZ;

for in the fig. $MN = MA \text{ tan. } A$

$=$ mer. diff. lat. . tan. course

but, form. [3] p. 122, diff. long. = mer. diff. lat. . tan. course

$\therefore$ MN = diff. long.

(9.) In working, therefore, examples in Navigation we see that the terms true diff. lat., departure, mer. diff. lat., diff. long., and course, may be correctly represented by the several parts of two similar right-angled plane triangles, and thus (except in the case when the course is 90° or parallel to the equator [p. 123]) the rules in Navigation can be made to depend on the solution of a right-angled plane triangle.

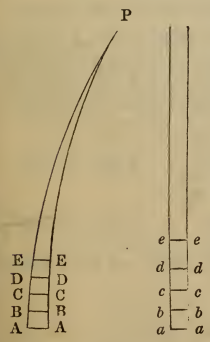
(10.) Before we deduce rules from the preceding

formulæ and principles, we will explain the construction of two tables made use of in this part of navigation. These are:

1. The Table of Meridional Parts.
2. The Traverse Table.

*Construction of the Table of Meridional Parts.*

We have seen (p. 30) that on a Mercator's chart the surface of the earth is projected into a plane, such that all the meridians become parallel straight lines, and the true difference of latitude between any two places is projected into the meridional difference of latitude; the true diff. lat. having the same proportion to the (projected or) meridional difference of latitude as the departure has to the difference of longitude into which it is projected. The distance (in miles) of any point on the chart from the equator is called the *meridional parts* for that latitude.




Let the arc AE of the meridian (the lat. of point E) be projected in the manner described above into the line *ae*. Then *ae* (in minutes or nautical miles) is called the meridional parts for the lat. E. Similarly, if AD be projected into *ad*, the line *ad* is the meridional parts for lat. D; and *ed*, the difference of the meridional parts for latitudes E and D, is the meridional difference of lat. for E and D.

The value of *ae*, the meridional parts for E, may be found correctly by means of the Integral Calculus. It may be obtained approximately thus:

Draw the meridians PA, PA, and parallels BB, CC, &c. sufficiently near each other, that the quadrilateral surfaces thus formed may be considered, without any practical error, as *planes;* this may be done if the arcs AB, BC, &c. be not greater than 1 minute; then the quadrilateral surfaces being projected into similar ones on the chart, as into *abba*, &c. the sides *ab bb* will have the same proportion to each other as AB BB; that is,

$$\frac{ab}{bb} = \frac{\text{AB}}{\text{BB}} \text{ or } \frac{ab}{\text{AB}} = \frac{bb}{\text{BB}} = \frac{\text{AA}}{\text{BB}} = \frac{1}{\text{cos. lat. B}} \quad \text{(p. 124)}$$

$$= \text{sec. lat. B.}$$

$$\therefore ab = \text{AB . sec. lat. B}$$

$$\text{similarly, } bc = \text{BC . sec. lat. C}$$

$$cd = \text{CD . sec. lat. D}$$

$$de = \text{DE . sec. lat. E}$$

Suppose now $\text{AB} = \text{BC} = \text{CD} = \text{DE} = 1'$

$\therefore ab + bc + cd + de$, or $ae = \text{sec.}\,1' + \text{sec.}\,2' + \text{sec.}\,3' + \text{sec.}\,4'$

A nearer approximation to the value of the meridional parts for 4 minutes would be obtained by taking the parts AB, BC, &c. each equal to 1″, for then the meridional parts for 4′ would be found from the expression,

mer. parts for $4' = \text{sec.}\,1'' + \text{sec.}\,2'' \ldots + \text{sec.}\,3'\,59''$.

Hence generally, for any lat. $l$,

mer. parts for $l^\circ = \text{sec.}\,1'' + \text{sec.}\,2'' \ldots + \text{sec.}\,(l^\circ - 1'')$

and in this manner was the first table of meridional parts constructed.

Ex. Having given the meridional parts for 70° (calculated as above or otherwise), find the meridional parts for 70° 10′.

M. P. for 70° 10′ = M. P. for 70° + sec. 70° + sec. 70° 1′. . . + sec. 70° 9′

| | | | |
|---|---|---|---|
| log. sec. 70° 0′ | .. 0·465948 | nat. sec. | 2·9238 |
| .. sec. 70 1 | .. 0·466296 | .. | 2·9262 |
| .. sec. 70 2 | .. 0·466643 | .. | 2·9285 |
| .. sec. 70 3 | .. 0·466991 | .. | 2·9308 |
| .. sec. 70 4 | .. 0·467339 | .. | 2·9332 |
| .. sec. 70 5 | .. 0·467688 | .. | 2·9355 |
| .. sec. 70 6 | .. 0·468037 | .. | 2·9379 |
| .. sec. 70 7 | .. 0·468386 | .. | 2·9403 |
| .. sec. 70 8 | .. 0·468735 | .. | 2·9426 |
| .. sec. 70 9 | .. 0·469085 | .. | 2·9450 |
| | | | 29·3438 |

mer. parts for 70° . . . . . 5965·92

∴ mer. parts for 7° 10′ . . . . 5995·26

(11.) A more concise rule for finding the meridional parts is as follows, derived from the investigation in the note below:*

* Let $l$ = lat. of D, fig. (p. 127).

M = corresponding angular measure of *ad*, the meridional parts for $l$.

$dl$, $d\mathrm{M}$, the contemporary increments of $l$ and M, as the angular measures of DE, *de* in the figure.

Now, as above, $\frac{de}{\mathrm{DE}} = \frac{dd}{\mathrm{DD}} = \frac{\mathrm{AA}}{\mathrm{DD}} = \sec. l$

$$\text{or } \frac{d\mathrm{M}}{dl} = \sec. l$$

$$\therefore \quad \mathrm{M} = \int \sec. l \, dl = \int \frac{\cos. l \, dl}{\cos.^2 l}$$

$$= \int \frac{\cos. l \, dl}{1-\sin.^2 l} = \int \frac{\frac{1}{2}\cos. l \, dl}{1+\sin. l} - \int \frac{-\frac{1}{2}\cos. l \, dl}{1-\sin. l}$$

$$= \tfrac{1}{2}\log_{\epsilon}(1+\sin. l) - \tfrac{1}{2}\log_{\epsilon}(1-\sin. l) + \text{cor.}\ (=0)$$

$$= \log_{\epsilon}\sqrt{\frac{1+\sin. l}{1-\sin. l}} = \log_{\epsilon}\sqrt{\frac{1+\cos. l_1}{1-\cos. l_1}} \quad (\text{if } l_1 = 90° - l)$$

*Rule for computing the Meridional Parts.*

Diminish the log. cotangent of half the colatitude by 10. Find the logarithm of the remainder, and add thereto the constant log. 3·8984895. The result will be the logarithm of the meridional parts (in minutes) for the given latitude.

EXAMPLES.

Required the meridional parts for latitudes 70° and 70° 10′.

| [1]. | | [2]. | |
|---|---|---|---|
| log. cot. 10° . . | 0·753681 | log. cot. 9° 55′ . | 0·757390 |
| log. ·753681 . . | $\overline{1}$·8771876 | log. ·75739 . . . | $\overline{1}$·8793200 |
| const. log. . . . . | 3·8984895 | const. log. . . . . | 3·8984895 |
| log. mer. parts | 3·7756771 | log. mer. parts | 3·7778095 |
| ∴ mer. parts . . . | 5965·92 | ∴ mer. parts . | 5995·28 |

*Construction of the Traverse Table.*

(12.) This table contains the true difference of latitude (diff. lat.) and departure (dep.) corresponding to

---

$$= \log_{\epsilon} \sqrt{\frac{2\cos.^2 \tfrac{1}{2} l_1}{2\sin.^2 \tfrac{1}{2} l_1}} = \log_{\epsilon} \cot. \tfrac{1}{2} l_1$$

Reducing to common logarithms (Part II. p. 108),

$$M = 2{\cdot}3025851 \log_{10} \cot. \tfrac{1}{2} l_1$$

$$\text{Now } M = \frac{\text{arc}}{\text{rad.}} = \frac{ad \text{ (in min.)}}{\text{rad. (in min.)}}$$

$$= \frac{\text{mer. parts for lat. } l}{57^{\circ}{\cdot}29577 \times 60}$$

$\therefore$ meridional parts $= 57^{\circ}{\cdot}29577 \times 60 \times 2{\cdot}3025851 \log_{10} \cot. \tfrac{1}{2} l_1$

In logarithms,

log. mer. parts for lat. $l = 3{\cdot}8984895 + \log_{10} (\log_{10} \cot. \tfrac{1}{2} \text{colat.} - 10)$

which, in words, is the rule given above.

every course from 0° to 90°, and for every distance from 1 mile to about 300. It is constructed as follows: a course and distance being assumed, the true difference of latitude and departure are computed for that course and distance by fig. p. 125, or by the formulæ [1] and [2], p. 122, namely:

true diff. lat. = dist. . cos. course
dep. = dist. . sin. course.

Ex. Given the course 25° and distance 26 miles; compute diff. lat. and dep.

true diff. lat = 26 . cos. 25°
dep. = 26 . sin. 25°

| | | | |
|---|---|---|---|
| log. 26 ..... | 1·414973 | log. 26 .... | 1·414973 |
| log. cos. 25° ..... | 9·957276 | log. sin. 25° | 9·625948 |
| log. true diff. lat. . | 1·372249 | log. dep. . . | 1·040921 |
| ∴ true diff. lat. . . . | 23·56 | ∴ dep. . . . | 10·99 |

When the difference of latitude and departure are computed up to 45°, the diff. lat. and dep. for courses above 45° may be found by interchanging the titles to the columns. Thus, to find the diff. lat. and dep. for 65° and distance 26 miles,

T. D. lat. for 65° = 26 . cos. 65° = 26 . sin. 25° = dep. for 25°
dep. for 65° = 26 . sin. 65° = 26 . cos. 25° = diff. lat. for 25°

Hence the diff. lat. and dep. for any course, are the dep. and diff. lat. respectively of the complement of that course. The difference of latitude and departure being computed in this manner, the quantities are tabulated in a form similar to the one on the other side:

| Distance 26 Miles. | | | |
|---|---|---|---|
| Course. | Diff. lat. | Dep. | Course. |
| 1° | | | 89° |
| 2° | | | 88 |
| . | | | . |
| . | | | . |
| . | | | . |
| 25° | 23·56 | 10·99 | 65 |
| . | | | . |
| . | | | . |
| . | | | . |
| . | | | . |
| . | | | . |
| 45° | | | 45 |
| Course. | Dep. | Diff. lat. | Course. |

We will here insert the usual preliminary or subordinate rules in navigation, before we shew the practical application of the preceding problems and tables.

## PRELIMINARY RULES IN NAVIGATION.

*Rule* (*a*).

*To find the true difference of latitude,* having given the latitude from and latitude in.*

(1.) When latitude from and latitude in have *like names,* that is, are both north or both south.

Under the latitude from, put down the latitude in, take the difference and reduce the same to minutes; place N. or S. against the result according as the latitude in is north or south of the latitude from; the remainder is the *true difference of latitude.*

(2.) When latitude from and latitude in have *unlike names,* that is, one north and the other south.

Take the sum of the two latitudes, reduce it to minutes, and attach N. or S. thereto according as the latitude in is north or south of the latitude from; the result is the *true difference of latitude.*

EXAMPLES.

1. Find the true difference of latitude, having given latitude from = 42° 10′ N., and latitude in 50° 48′ N.

lat. from 42° 10′ N.
lat. in 50 48 N.

8 38
60

T. D. lat. 518 N.

* The latitude of the place left is called the latitude *from,* the latitude of the place arrived at is called the latitude *in.*

2. Find the true difference of latitude, having given latitude from 3° 42′ N., and latitude in 2° 40′ S.

lat. from 3°42′ N.
lat. in 2 50 S.

6 32
60

T. D. lat. 392 S.

Find the true difference of latitude in each of the following examples:

| | Lat. from | Lat. in | Answers. |
|---|---|---|---|
| 3. | 33°42′ N. | 40°40′ N. | T. D. lat. = 418 N. |
| 4. | 40 40 N. | 33 42 N. | .. = 418 S. |
| 5. | 3 42 S. | 1 40 N. | .. = 322 N. |
| 6. | 3 8 S. | 14 42 S. | .. = 694 S. |
| 7. | 68 48 N. | 38 30 N. | .. =1818 S. |
| 8. | 14 14 N. | 0 0 | .. = 854 S. |

*Rule* (*b*).

*To find the meridional difference of latitude,* having given the latitude from and latitude in.

Take the meridional parts for the two latitudes from the table of meridional parts; subtract, if the names be alike, and add if the names be unlike; the result is the *meridional difference of latitude,* N. or S. being attached thereto according as the latitude in is north or south of latitude from.

EXAMPLES.

9. Find the meridional difference of latitude, having given latitude from 42°10′ N., and latitude in 50°48′ N.

lat. from 42°10′ N. mer. parts 2795·2 N.
lat. in 50 48 N. mer. parts 3549·8 N.

mer. diff. lat. 754·6 N.

10. Find the meridional difference of latitude, having given latitude from 3° 42′ N., and latitude in 7° 32′ S.

| | | |
|---|---|---|
| lat. from | 3° 42′ N. | mer. parts 222·2 N. |
| lat. in | 7 32 S. | mer. parts 453·3 S. |
| | | mer. diff. lat. 675·5 S. |

Find the meridional difference of latitude in each of the following examples:

| | Lat. from | Lat. in | | Answers. |
|---|---|---|---|---|
| 11. | 34° 42′ N. | 33° 15′ N. | M.D. lat. = | 104·9 S. |
| 12. | 14 14 N. | 30 14 N. | .. = | 1041·7 N. |
| 13. | 84 10 N. | 80 30 N. | .. = | 1681·5 S. |
| 14. | 2 8 S. | 3 10 N. | .. = | 318·1 N. |
| 15. | 4 5 N. | 4 5 S. | .. = | 490·4 S. |
| 16. | 0 0 | 2 45 N. | .. = | 165·1 N. |

*Rule* (*c*).

*To find the middle latitude,* having given the latitude from and latitude in.

The names being supposed to be *alike,* that is, both north or both south.

Add together the two latitudes, and take half the sum; the result is the middle latitude.

When the names are unlike, the mid. lat. (which is seldom required but for obtaining the departure) should be found by means of a table; but in this case it may perhaps be as well to avoid the use of the middle latitude in any of the common problems in navigation.

EXAMPLES.

17. Find the middle latitude, having given latitude from 3° 42′ N., and latitude in 13° 52′ N.

lat. from 3° 42′ N.
lat. in 13 52 N.

2)17 34

mid. lat. 8 47 N.

Find the middle latitude in each of the following examples :

| | Lat. from | Lat. in | Answers. |
|---|---|---|---|
| 18. | 38° 42′ N. | 30° 30′ N. | mid. lat. 34° 36′ N. |
| 19. | 62 17 S. | 62 30 S. | .. 62 23½ S. |

*Rule (d).*

*To find the difference of longitude,* have given the longitude from and longitude in.

(1.) When the longitude from and longitude in have *like names;* that is, are both east or both west.

Under longitude from put longitude in, take the difference, and reduce the same to minutes; place E. or W. against the remainder according as the longitude in is east or west of longitude from; the remainder will be the difference of longitude.

(2.) When the longitude from and longitude in have *unlike names,* that is, one east and the other west.

Take the sum of the two longitudes, reduce it to minutes, and attach E. or W. thereto according as the longitude in is east or west of the longitude from; the result is the true difference of longitude.

Note. If the difference of longitude found by this rule exceed 180° it must be subtracted from 360°, and the remainder brought into minutes must be considered the difference of longitude, with the contrary letter attached to it.

EXAMPLES.

20. Find the difference of longitude, having given the longitude from = 110° 42′ W., and longitude in 100° 42′ W.

long. from 110° 42′ W.
long. in 100 42 W.

10 0
60

diff. long. 600 E.

21. Find the difference of longitude, having given long. from 12° 10′ E., and long. in 2° 45′ W.

long. from 12° 10′ E.
long. in 2 45 W.

14 55
60

diff. long. 895 W.

Find the difference of longitude in each of the following examples:

| | Long. from | Long. in | Answers. |
|---|---|---|---|
| 22. | 33° 40′ E. | 40° 10′ E. | Diff. long. 390 E. |
| 23. | 104 0 W. | 110 30 W. | .. 390 W. |
| 24. | 2 45 W. | 3 30 E. | .. 375 E. |
| 25. | 0 0 | 4 10 W. | .. 250 W. |
| 26. | 3 10 W. | 3 10 E. | .. 380 E. |
| 27. | 179 0 E. | 179 0 W. | .. 120 E. |

*Rule* (*e*).

*To find the latitude in*, having given the latitude from and true difference of latitude.

(1.) When the latitude from and true difference of latitude have *like names*.

To the latitude from, *add* the true difference of latitude (turned into degrees and minutes, if necessary); the sum will be the latitude in, of the same name as the latitude from.

(2.) When the latitude from and true difference of latitude have *unlike names*.

Under the latitude from put the true difference of latitude (in degrees and minutes, if necessary); take the less from the greater; the remainder, marked with the name of the greater, is the latitude in.

EXAMPLES.

28. Find the latitude in, having given the latitude from 42° 30′ N., and true difference of latitude 342′ N.

| | |
|---|---|
| 60)342′ N. | lat. from 42° 30′ N. |
| 5° 42′ N. | T. D. lat. 5 42 N. |
| | lat. in .. 48 12 N. |

29. Find the latitude in, having given the latitude from 2°40′ S., and true difference latitude 342′ N.

| | |
|---|---|
| 60)342′ N. | lat. from 2°40′ S. |
| 5°42′ N. | T. D. lat. 5 42 N. |
| | lat. in 3 2 N. |

Find the latitude in, in each of the following examples:

| | Lat. from | T. D. lat. | Answers. |
|---|---|---|---|
| 30. | 30°10′ N. | 182′ N. | Lat. in 33°12′ N. |
| 31. | 3 2 S. | 190 N. | .. 0 8 N. |
| 32. | 2 48 S. | 368 N. | .. 3 20 N. |
| 33. | 2 48 S. | 288 N. | .. 2 0 N. |
| 34. | 4 48 N. | 288 S. | .. 0 0 |
| 35. | 0 10 N. | 228 N. | .. 3 58 N. |

### *Rule* (*f*).

*To find the longitude in,* having given the longitude from and the difference of longitude.

(1.) When the longitude from and diff. long. have *like names.*

To the long. from, *add* diff. long. (turned into degrees if necessary); the sum will be long. in, of the same name as long. from.

(2.) When the long. from and diff. long. have *unlike names.*

Under long. from put diff. long. (in degrees and minutes, if necessary); take the less from the greater; the remainder, marked with the name of the greater, is the long. in.

NOTE. If the long. in, found as above, exceed 180°, subtract it from 360°, and attach to the remainder the contrary name to the one directed in the Rule.

EXAMPLES.

36. Find the long. in, having given long. from 38°42′ W., and diff. long. 384·5′ W.

60)384·5
6° 24·5′ W.

long. from 38°42′ W.
diff. long. 6 24·5 W.
long. in 45 6·5 W.

Find the longitude in, in each of the following examples:

| | Long. from | Diff. long. | Answers. |
|---|---|---|---|
| 37. | 62°32′ E. | 1000·5′ W. | long. in 45°51·5′ E. |
| 38. | 2 30 E. | 126·6 E. | .. 4 36·6 E. |
| 39. | 3 40 W. | 220·0 E. | .. 0 0 |
| 40. | 0 0 | 100·4 W. | .. 1 40·4 W. |
| 41. | 179 59 W. | 2·0 W. | .. 179 59·0 E. |

## THE COMPASS AND LOG-LINE.

The Compass card is represented on page 120; each quadrant is divided into eight equal parts, called *points;* each point therefore contains 11° 15′. The names of the points are formed as follows:

Middle point between N. and E. is formed by putting these letters together, thus . . . N.E.
Middle point between N. and N.E. is formed thus N.N.E.
Middle point between E. and N.E. is formed thus E.N.E.
One point from N. towards E. is N. by E. or . N.b.E.
..... E. .... N. is E. by N. or . E.b.N.
..... N.E. .... N. is N.E. by N. or N.E.b.N.
..... N.E. .... E. is N.E. by E. or N.E.b.E.

The points of the compass are frequently referred to with respect to their position to the right or left of the cardinal point towards which the spectator is looking; thus, N.E. is said to be 4 points to the right of north; E.b.N. is 7 points to the right of north. Each point, moreover, is subdivided into quarter points, and named from the adjacent points; thus, $2\frac{1}{2}$ points to the right of north is N.N.E.$\frac{1}{2}$E.; $7\frac{3}{4}$ points to the right of north is E.b.N.$\frac{3}{4}$E. or E.$\frac{1}{4}$N.

The other three quadrants are divided and referred to in a similar manner.

Attached to the compass, and coinciding with the line N.S. is a magnetic bar of steel, by means of which the card, when balanced on a fine point near its center, will indicate the compass bearing or direction of any object beyond it. Thus, the compass being placed near

the helm, the bearing of the ship's head is seen at once, and the direction in which the ship is steered is readily noted.

*The Log-line.*

The log is a flat piece of thin wood of a quadrantal form, loaded in the circular side with lead sufficient to make it swim upright in the water; to this is fastened a line about 150 fathoms long, called the *log-line,* which is divided into certain spaces called *knots;* the length of each knot is supposed to be the same part of a nautical mile (6120 feet) that half a minute is of an hour, hence 1 knot $= \frac{6120}{120} = 51$ feet. If, therefore, 1 knot runs out in half a minute (shewn by a half-minute glass), the rate of the ship is supposed to be 1 mile an hour; if 2 knots, the rate is 2 miles an hour, and so on. The length of the knot is very rarely so much as 51 feet, and the hour-glass used is not always a half-minute glass; various modifications of the two instruments are made, to render this method of measuring the ship's way tolerably correct; these will be more clearly seen in the use of the instruments themselves.

*Correcting Courses.*

*Three* corrections are sometimes necessary to be applied to the course steered by compass, to reduce it to the true course; and the converse. These are called:

(1.) The variation of the compass.
(2.) The deviation of the compass.
(3.) The leeway.

(1.) *The Variation of the Compass.*

The magnetic needle seldom points to the true north.

Its deflection to the east or west of the true north is called the *variation of the compass;* it is different in different places, and it is also subject to a slow change in the same place. The variation of the compass is ascertained at sea by observing the magnetic bearing of the sun when in the horizon, or at a given altitude above it. From this observation the true bearing is found by rules given in nautical astronomy. The difference between the true bearing and the observed bearing by compass determines this correction.

The method of correcting the course for variation will be more readily understood by means of an example.

Suppose the variation of the compass is found to be 2 points to the east, that is, the needle is directed 2 points to the right of the north point of the heavens; then the N.N.W. point of the compass card will evidently point to the true north, and every other point on the card will be shifted round 2 points. If, therefore, a ship is sailing *by compass* N.N.W., or, as it is expressed, the compass course is N.N.W., her true course will be north; that is, 2 *points to the right of the compass course.* In a similar manner it may be shewn that, when the variation is 2 points westerly, the true course will be 2 *points to the left of compass course.* Hence this rule:

*Rule* (*g*).

*To find the true course,* the compass course being given.

Easterly variation allow to the right.

Westerly ..... left.

From the preceding considerations it will be easy to deduce the converse rule, namely:

*Rule* (*h*).

*To find the compass course,* the true course being given.

Easterly variation allow to the left.
Westerly ..... right.

EXAMPLES TO RULES (*g*) AND (*h*).

42. Find the true course, having given the compass course N.W.$\frac{1}{2}$W. and variation 3$\frac{1}{4}$W.

| | pts. | qrs. | |
|---|---|---|---|
| Compass course | 4 | 2 | left of N. |
| variation ..... | 3 | 1 | left.* |
| true course .... | 7 | 3 | left of N. = W.$\frac{1}{4}$N. |

43. Find the compass course, having given the true course W.$\frac{1}{4}$N. and variation 3$\frac{1}{4}$W.

| | pts. | qrs. | |
|---|---|---|---|
| True course ... | 7 | 3 | left of N. |
| variation ..... | 3 | 1 | right |
| compass course | 4 | 2 | left of N.=N.W.$\frac{1}{2}$W. |

Find the true course in each of the following examples:

| | Compass course. | Var. | Answers. |
|---|---|---|---|
| 44. | N.N.E. | 2$\frac{1}{4}$W. | N.$\frac{1}{4}$W. |
| 45. | N.W. | 1$\frac{3}{4}$E. | N.N.W.$\frac{1}{4}$W. |
| 46. | S.W.$\frac{3}{4}$W. | 1$\frac{1}{2}$E. | W.S.W.$\frac{1}{4}$W. |
| 47. | S. | 2W. | S.S.E. |
| 48. | W. | 2$\frac{1}{2}$E. | N.W.b.W.$\frac{1}{2}$W. |

* When names are alike, (that is, both left or both right,) *add:* when unlike, *subtract,* marking remainder with the name of the greater.

Find the compass course in each of the following examples:

| | True course. | Var. | Answers. |
|---|---|---|---|
| 49. | N.N.E.$\frac{1}{2}$E. | $\frac{1}{4}$W. | N.N.E.$\frac{3}{4}$E. |
| 50. | N. | 1$\frac{1}{2}$E. | N.b.W.$\frac{1}{2}$W. |
| 51. | S.S.W. | 2W. | S.W. |
| 52. | S.W. | 0 | S.W. |
| 53. | N.b.W.$\frac{1}{4}$W. | 1$\frac{1}{4}$W. | N. |

### (2.) *Deviation of the Compass.*

This correction of the compass arises from the effect of the iron on board ship on the magnetic needle, in deflecting it to the right or left of the magnetic meridian. The increased quantity of iron used in ships, especially in steamers, has caused this correction to be attended to now more than formerly, as its effects and magnitude have become more perceptible. The amount of the deviation arising from this local cause varies as the mass of iron changes its position with respect to the compass. When a fore and aft line coincides with the direction of the magnetic meridian, the iron in the ship may be supposed to be nearly equally distributed on both sides of the needle, and its effect in deflecting the needle may be inappreciable. In other positions of the ship with respect to the magnetic meridian, the iron may produce a sensible deflection of the needle; and this deflection or deviation will in general be the greatest when the ship's head points to the east or west.

Various methods are used to determine this correction. The one usually adopted is to place a compass on shore, where it may be beyond the influence of the iron of the ship, or any other local disturbing force, and to take the bearing of the ship's compass, or some

object in the same direction therewith; at the same time, the observer on board takes the bearing of the shore compass; then, if 180° be added to the bearing of the shore compass, so as to bring it round to the opposite point, the difference between the result and the bearing of ship's compass will be the amount of the deviation of the compass for that position of the ship. The ship is then swung round one or two points, and a similar observation made; and thus the local deviation found for a second position of the ship. This being repeated for every point or two points of the compass, the deviation is thus known for all positions of the ship. A table, similar to the one below, is then formed, and the courses corrected for this deviation by the following rules; which resemble those already given for correcting for variation.

*Deviation of Compass of H.M.S. ———, for given positions of the Ship's Head:*

| Direction of ship's head. | Deviation of compass. | | Direction of ship's head. | Deviation of compass. | |
|---|---|---|---|---|---|
| | | nearly | | | nearly |
| N. | E. 2° 45′ | or ¼ pt. | S. | W. 3° 0′ | or ¼ pt. |
| N.b.E. | E. 4 57 | or ½ .. | S.b.W. | W. 4 20 | or ½ .. |
| N.N.E. | E. 7 30 | or ¾ .. | S.S.W. | W. 5 0 | or ½ .. |
| N.E.b.N. | E. 9 0 | or ¾ .. | S.W.b.S. | W. 6 7 | or ½ .. |
| N.E. | E. 10 0 | or ¾ .. | S.W. | W. 7 0 | or ½ .. |
| N.E.b.E. | E. 10 55 | or 1 .. | S.W.b.W. | W. 7 27 | or ½ .. |
| E.N.E. | E. 10 40 | or 1 .. | W.S.W. | W. 7 50 | or ¾ .. |
| E.b.N. | E. 9 55 | or ¾ .. | W.b.S. | W. 8 20 | or ¾ .. |
| E. | E. 8 50 | or ¾ .. | W. | W. 8 50 | or ¾ .. |
| E.b.S. | E. 7 15 | or ½ .. | W.b.N. | W. 8 10 | or ¾ .. |
| E.S.E. | E. 5 35 | or ½ .. | W.N.W. | W. 6 50 | or ½ .. |
| S.E.b.E. | E. 3 40 | or ¼ .. | N.W.b.W. | W. 5 40 | or ½ .. |
| S.E. | E. 1 50 | or ¼ .. | N.W. | W. 4 50 | or ½ .. |
| S.E.b.S. | E. 0 20 | or 0 .. | N.W.b.N. | W. 3 20 | or ¼ .. |
| S.S.E. | W. 0 56 | or 0 .. | N.N.W. | W. 1 40 | or 0 .. |
| S.b.E. | W. 2 20 | or ¼ .. | N.b.W. | E. 1 10 | or 0 .. |

*Rule* (*i*).

*To find the true course,* having given the compass course and the deviation.

Easterly deviation allow to the right.
Westerly ..... left.

EXAMPLES.

54. Correct the compass course W.b.S. for deviation $\frac{3}{4}$ W. (known from table, p. 145).

| | pts. | qrs. | |
|---|---|---|---|
| Compass course | 7 | 0 | right of S. |
| deviation ..... | 0 | 3 | left |
| true course .... | 6 | 1 | right of S. |

or W.S.W.$\frac{1}{4}$W.

55. Correct the compass course N.W.$\frac{1}{2}$W. for deviation $\frac{1}{2}$W. (from deviation table, p. 145), and also for variation of compass $3\frac{1}{4}$W.

| | pts. | qrs. | |
|---|---|---|---|
| Compass course .... | 4 | 2 | l. N. |
| deviation .... | 0 | 2 | l. |
| variation .... | 3 | 1 | l. |
| | 3 | 3 | l. |
| true course..... | 8 | 1 | l. N. |
| | 16 | | |

or true course ..... 7 3 r. S. = W.$\frac{1}{4}$S.

Find the true course in each of the following examples, by correcting for deviation from table, p. 145, and for variation.

| | Compass course. | Var. | Answers. |
|---|---|---|---|
| 56. | N.N.E. | $2\frac{1}{4}$W. | N.$\frac{1}{2}$E. |
| 57. | N.W. | $1\frac{3}{4}$E. | N.N W.$\frac{3}{4}$W. |

| | Compass course. | Var. | Answers. |
|---|---|---|---|
| 58. | S.W.$\frac{3}{4}$W. | $1\frac{1}{2}$E. | S.W.b.W.$\frac{3}{4}$W. |
| 59. | S. | 2W. | S.S.E.$\frac{1}{4}$E. |
| 60. | W. | $2\frac{1}{2}$E. | W.N.W.$\frac{1}{4}$W. |
| 61. | W.$\frac{3}{4}$N. | $1\frac{1}{2}$W. | W.S.W.$\frac{1}{2}$W. |

### *Rule* (*k*).

*To find the compass course,* having given the true course and deviation.

Easterly deviation allow to the left.

Westerly ..... right.

NOTE. The true course should first be corrected for variation (if any) by Rule (*h*), (which is similar to the above), so as to get a compass course nearly, and then this course for deviation, from table, p. 145.

EXAMPLES.

62. Required the compass course, the true course being W.S.W.$\frac{1}{4}$W., variation 0, and deviation $\frac{3}{4}$W. (see table).

| | pts. | qrs. | |
|---|---|---|---|
| True course .. | 6 | 1 | r. S. |
| deviation ..... | 0 | 3 | r. |
| compass course | 7 | 0 | r. S., or W.b.S. |

63. Required the compass course, the true course being S.W., variation of compass $2\frac{1}{4}$E., and deviation as in table, p. 145.

| | pts. | qrs. | |
|---|---|---|---|
| True course..... | 4 | 0 | r. S. |
| variation ..... | 2 | 1 | l. |
| compass course nearly | 1 | 3 | r. S., or S.b.W.$\frac{3}{4}$W. |
| deviation..... | 0 | 2 | r. |
| compass course .... | 2 | 1 | r. S. = S.S.W.$\frac{1}{4}$W. |

Required the compass course in each of the following examples (for deviation, see table, p. 145):

| | True course. | Var. | Answers. |
|---|---|---|---|
| 64. | N.$\frac{1}{2}$E. | 2$\frac{1}{4}$W. | N.N.E. |
| 65. | N.N.W.$\frac{3}{4}$W. | 1$\frac{3}{4}$E. | N.W. |
| 66. | S.W.b.W.$\frac{3}{4}$W. | 1$\frac{1}{2}$E. | .W .$\frac{3}{4}$W. |
| 67. | S.S.E.$\frac{1}{4}$E. | 2W. | S. |
| 68. | W.N.W.$\frac{1}{4}$W. | 2$\frac{1}{2}$E. | W. |
| 69. | W.S.W.$\frac{1}{2}$W. | 1$\frac{1}{2}$W. | W.$\frac{3}{4}$N. |

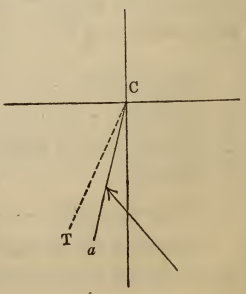
### (3.) *Leeway.*

This correction is the angle which the ship's track makes with the direction of a fore and aft line: it arises from the action of the wind on the sails, &c. not only impelling the ship forwards, but pressing against it sideways, so as to cause the actual course made to be to *leeward* of the apparent course, as shewn by the fore and aft line. The amount of leeway differs in different ships, depending on their construction, on the sails set, the velocity forwards, and other circumstances. Experience and observation, therefore, usually determine the amount of leeway to be allowed.

### *Rule* (*l*).

The method of correcting for leeway will be best seen by the following example:

Suppose the apparent course is S.S.W.$\frac{1}{2}$W., and leeway 2 points, the wind being S.E., required the correct course.




Draw two lines at right angles to each other towards the cardinal points of compass, and a line, as C*a*, to represent

(roughly) the course of the ship, and another to represent the direction of the wind (S.E.); then it will be seen that the corrected course, as CT, will be to the *right* of the apparent course; the observer being always supposed to be at the center, C, and looking towards the cardinal point from whence the course is measured; hence

| | pts. | qrs. | |
|---|---|---|---|
| Apparent course ... | 2 | 2 | r. S. |
| leeway ..... | 2 | 0 | r. |
| corrected course ... | 4 | 2 | r. S. = S.W.$\frac{1}{2}$W. |

EXAMPLES.

Correct the following courses for leeway, so as to find the true courses:

| | Apparent course. | Wind. | Leeway. | Answers. |
|---|---|---|---|---|
| 70. | N.N.E. | W.N.W. | $1\frac{1}{2}$ | N.E.$\frac{1}{2}$N. |
| 71. | N.W. | N.N.E. | 2 | W.N.W. |
| 72. | E.S.E. | S. | $2\frac{1}{2}$ | E.$\frac{1}{2}$N. |
| 73. | E. | N.b.E. | $\frac{3}{4}$ | E.$\frac{3}{4}$S. |

Correct the following compass courses for deviation, variation, and leeway, so as to find the true courses. The deviation is found in table, p. 145, and the variation of compass is supposed to be in each example $2\frac{1}{2}$ W.

| | Course. | Wind. | Leeway. | Answers. |
|---|---|---|---|---|
| 74. | N.W.$\frac{1}{4}$W. | W.S.W. | $2\frac{1}{2}$ | N.W.$\frac{3}{4}$W. |
| 75. | S.E.$\frac{1}{2}$E. | E.N.E. | $2\frac{1}{4}$ | S.E.$\frac{1}{2}$E. |
| 76. | W.$\frac{1}{4}$S. | S.S.W. | 2 | W.S.W.$\frac{1}{2}$W. |
| 77. | N.$\frac{3}{4}$W. | W.b.N. | $1\frac{1}{2}$ | N.b.W.$\frac{1}{2}$W. |

These examples may be worked out in the following manner:

| | | qrs. | pts. | |
|---|---|---|---|---|
| Ex. 74. | Compass course.... | 4 | 1 | l. N. |
| | dev..... | 0 | 2 | l. |
| | var..... | 2 | 2 | l. |
| | | 3 | 0 | l. |
| | | 7 | 1 | l. N. |
| | leeway ..... | 2 | 2 | r. |
| | true course..... | 4 | 3 | l. N. = N.W.$\frac{3}{4}$W. |

AN EXAMINATION PAPER IN THE PRECEDING PRINCIPLES.

(1.) Give definitions of the following terms used in navigation: course, distance, true difference of latitude, difference of longitude, departure, and meridional difference of latitude.

(2.) Prove the four fundamental formulæ in navigation from which the rules are derived.

(3.) Prove that the terms, distance, true diff. lat., dep., and course may be correctly represented by the three sides and an angle of a right-angled plane triangle; and that the mer. diff. lat. and diff. long. may be represented by two sides of a triangle which is similar to the above.

(4.) The arc TS (p. 29) of a parallel of lat., drawn through the middle lat., is not exactly equal to the departure between A and F. Shew that the latitude $x$, in which the parallel is equal to the departure, may be found from the expression, cos. lat. $x = \frac{\text{T. D. lat.}}{\text{M. D. lat.}}$

(5.) Describe the Mercator's chart, and shew that in such a chart the course from one place to another is the angle which a straight line (as the edge of a ruler) passing through the two places makes with the meridian.

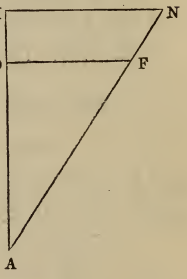
# RULES IN NAVIGATION.

## *Rule I.* (*Mercator's method*).

*To find the course and distance* from one place to another, having given the latitudes and longitudes of the two places.

(1.) Find the T. D. lat., diff. long., and M. D. lat.

(2.) Then if A and F represent the two places, there are given AO the T. D. lat., MN the diff. long., and AM the M. D. lat.; to find the angle A *the course*, and AF *the distance*.




(3.) To find course.

$$\text{tan. course} = \frac{\text{MN}}{\text{AM}} = \frac{\text{diff. long.}}{\text{M.D. lat.}}$$

(4.) To find distance.

$$\text{dist.} = \text{T. D. lat.} \,.\, \text{sec. course.}$$

When tan. course is found by (3), take out sec. course from the same opening of tables.

## *Rule I.* (*in words*).

(1.) Find true difference latitude, meridional difference latitude, and difference longitude; reduce true difference latitude and difference longitude to minutes, attaching thereto the proper letters. Rules (*a*), (*b*), (*d*).

(2.) *To find course.* From the log. difference longi-

tude (increased by 10), subtract the log. mer. diff. latitude; the remainder is the log. tan. course, which find in the tables, and place before it the letter of the true difference latitude, and after it the letter of the difference longitude, to indicate the direction of course. At the same opening of the tables, take out the log. secant course.

(3.) *To find distance.* Add together log. secant course, and log. true difference latitude; the sum (rejecting 10 in the index) will be the log. distance, which find in the tables.

EXAMPLES.

78. Required the course and distance from A to B.

lat. A 45° 15′ N. long. A 35° 26′ W.
lat. B 47 10 N. long. B 32 15 W.

$$\left.\begin{aligned} \text{tan. course} &= \frac{\text{diff. long.}}{\text{M. D. lat.}} \\ \text{dist.} &= \text{T. D. lat.} \,..\, \text{sec. course.} \end{aligned}\right\} \text{fig. p. 151.}$$

lat. A .. 45° 15′ N. M.P.. . 3051·2 N. long. A 35° 26′ W.
lat. B .. 47 10 N. 3217·4 N. long. B 32 15 W.

1 55 M.D.lat. 166·2 N. 3 11
60 60

T.D. lat. 115 N. diff. long. 191 E.

log. diff. long. + 10, 12·281033
log. M. D. lat. .. 2·220631

log. tan. course .. 10·060402 .. log. sec. course 0·182767
∴ course N. 48° 58′ E. log. T. D. lat. 2·060698
log. dist. ... 2·243465
∴ distance 175′

Required also the compass course in the above example: var. of compass being 2 points W, and devia-

tion on account of local attraction, as in table (p. 145). See Rule (*k*).

True course ..... 48° 58′ r. N.

pts. qrs.
or 4 1 r. N.*

variation..... 2 0 r.

compass course nearly 6 1 r. N.=E.N.E.$\frac{1}{4}$E.

deviation..... 1 0 l.

∴ compass course.... 5 1 r. N.=N.E.b.E.$\frac{1}{4}$E.

Required the course and distance from A to B in each of the following examples, by Rule I. or Mercator's method.

| | Lat. from and lat. in. | Long. from and long. in. | Answers. Course and distance. |
|---|---|---|---|
| 79. | lat. A 49° 52′ S. | long. A 17° 22′ W. | course N. 26° 36′ E. |
| | .. B 42 13 S. | .. B 11 50 W. | dist... 513·3 miles. |
| 80. | lat. A 49 10 N. | long. A 29 17 W. | course N. 37° 48′ W. |
| | .. B 56 45 N. | .. B 39 5 W. | dist... 576 miles. |
| 81. | lat. A 50 48 N. | long. A 1 10 E. | course N. 41° 55′ W. |
| | .. B 52 35 N. | .. B 1 25 W. | dist... 143·8 miles. |
| 82. | lat. A 58 24 N. | long. A 4 12 W. | course N. 32° 34′ E. |
| | .. B 63 17 N. | .. B 2 13 E. | dist... 347·6 miles. |
| 83. | lat. A 2 37 N. | long. A 110 42 W. | course S. 75° 11′ W. |
| | .. B 0 0 | .. B 120 36 W. | dist... 614 miles. |
| 84. | lat. A 3 30 N. | long. A 33 40 E. | course S. 42° 31′ E. |
| | .. B 4 10 S. | .. B 40 42 E. | dist... 624 miles. |

Required also the compass courses in examples 82, 83, and 84, the variation of compass being 2 points E., and deviation as in table, p. 145. See Rule (*k*).

ANSWERS.

82. compass course .. N.$\frac{1}{2}$E. nearly.

83. .. .. .. S.W.b.W.$\frac{1}{4}$W. nearly.

84. .. .. .. E.S.E.$\frac{1}{4}$E. ,,

* Degrees are converted into points, or the converse, by means of the table for that purpose in the nautical tables.

### *Rule II.*

To find the latitude in and longitude in, having given the course from a given place, and distance.

In fig. (p. 151), let A be the place sailed from, and F the place arrived at; then we have given the angle A = course, and AF = distance, to find AO the T. D. lat., and MN the diff. long.

(1.) To find AO the T. D. lat.

T. D. lat. = dist. . cos. course.

(2.) Apply T. D. lat. thus found, to lat. from, and get lat. in. Find meridional parts for lat. from and lat. in, and thus determine AM the M. D. lat.

(3.) To find MN the diff. long.

diff. long. = M. D. lat. . tan. course.

(4.) Apply diff. long. to long. from, and thus get long. in.

### *Rule II.* (*in words*).

(1.) To find latitude in. Add together log. cos. course* and log. distance, the sum (rejecting 10 in the index) will be the log. true difference latitude, which find in the tables; reduce to degrees and minutes, and place the letter N. or S. against it, according as the course is northward or southward.

(2.) Apply true difference latitude to latitude from, so as to get the latitude in. (Rule *e*.)

(3.) To find longitude in. Take out the meridional parts for the two latitudes, and get the meridional difference latitude. (Rule *b*.)

(4.) Add together log. tangent course and log. me-

* Take out, at same opening of tables, log. tan. course, and place it a little to the right.

ridional difference latitude; the sum (rejecting 10 in the index) will be the log. difference longitude, which find in the tables; reduce to degrees and minutes, and place the letter E. or W. against it, according as the course is eastward or westward.

(5.) Apply difference longitude to longitude from, so as to get longitude in. (Rule *f*.)

EXAMPLES.

85. Sailed from A, N. 37° 10′ E., 472·6 miles; required the latitude and longitude in.

lat. A 27° 20′ N. long. A 25° 12′ E.

(1.) T. D. lat. = dist. . cos. course.

(2.) diff. long. = M. D. lat. . tan. course.

| | |
|---|---|
| log. cos. course 9·901394 | log. tan. course 9·879740 |
| log. dist. .... 2·674494 | log. M. D. lat. 2·641474 |
| log. T. D. lat. . 2·575888 | log. diff. long. 2·521214 |
| ∴ T. D. lat. 376·6′ | diff. long. 332·1′ |
| or 6° 17′ N. | or 5° 32′ E. |
| M. P. | |
| lat. from 27 20 N. . . 1706 N. | long. from 25 12 E. |
| lat. in . . 33 37 N. . . 2144 N. | long. in . . 30 44 E. |
| M.D. lat. 438 | |

Required the latitude and longitude in, by Rule II. or Mercator's method, in each of the following examples, having sailed from A as follows:

| | Course and dist. from A. | Lat. A. | Long. A. | Answers. Lat. in. | Long. in. |
|---|---|---|---|---|---|
| 86. | N. 26° 36′ E. 513·5′ | 49° 52′ S. | 17° 22′ W. | 42° 13′ S. | 11° 50′ W. |
| 87. | S. 48 58 W. 175·2 | 47 10 N. | 32 15 W. | 45 15 N. | 35 26 W. |
| 88. | N. 29 10 E. 373·4 | 52 10 N. | 17 32 W. | 57 36 N. | 12 15 W. |
| 89. | N. 31 4 W. 318·8 | 57 40 N. | 12 16 W. | 62 13 N. | 17 45 W. |
| 90. | S. 37 7 E. 370·0 | 70 14 S. | 25 30 E. | 75 9 S. | 38 5 E. |
| 91. | N. 47 47 E. 272·4 | 50 15 S. | 15 10 E. | 47 12 S. | 20 16 E. |

*Middle Latitude method.*

Approximate rules may be formed for finding the course and distance from one place to another, the latitude and longitude of the two places being given; and, conversely, the latitude and longitude in may be found, having given the course from a given place and distance, by combining formula [5] with formulæ [1] and [2], p. 122, 125.

*Rule III.* (*Middle Latitude method*).

To find the course and distance, having given the latitudes and longitudes of the two places.

(1.) Find T. D. lat., mid. lat., and diff. long.

(2.) Find departure.

In fig. p. 29, $\frac{TS}{UZ} = \text{cos. TU}$

or dep. (nearly) = diff. long. . cos. mid. lat.

(3). In fig. p. 151, given OF and AO, the dep. and T. D. lat., to find course and distance, thus:

$$\text{tan. course} = \frac{\text{dep.}}{\text{T. D. lat.}} = \frac{\text{diff. long. . cos. mid. lat.}}{\text{T. D. lat.}} \quad \text{nearly}$$

dist. = T. D. lat. . sec. course.

*Rule III.* (*in words*).

(1.) Find the true difference latitude, middle latitude, and difference longitude. (*a*), (*c*), (*d*).

(2.) To find the course. Add together log. cos. mid. lat. and log. diff. long., and from the sum subtract log. true difference latitude; the remainder is the log. tan. course, which find in the tables, and mark it with the same letters as the true difference latitude and difference longitude. From the same opening take out the log. secant of course.

(3.) To find distance. To the log. secant course just found, add the log. true difference latitude; the sum (rejecting 10 in index) will be the log. distance.

EXAMPLES.

92. Required the course and distance from A to B, by middle latitude method.

lat. A 50° 25′ N. long. A 27° 15′ W.
lat. B 47 12 N. long. B 30 20 W.

(1.) tan. course $= \frac{\text{diff. long.} \cdot \text{cos. mid. lat.}}{\text{T. D. lat.}}$

(2.) dist. = T. D. lat. . sec. course.

lat. A . . 50° 25′ N. . . . 50° 25′ N. long. A 27° 15′ W.
lat. B . . 47 12 N. . . . 47 12 N. long. B 30 20 W.
3 13 2)97 37 3 5
60 mid. lat. 48 48 60
T. D. lat. 193 S. diff. long. 185 W.
log. cos. mid. lat. 9·818681 log. sec. course 0·072849
log. diff. long. . . 2·267172 log. T. D. lat. 2·285557
12·085853 log. dist. . . . . 2·358406
log. T. D. lat. . . 2·285557 ∴ dist. 228·2′
log. tan. course 9·800296 ∴ course S. 32° 16′ W.

Required the course and distance from A to B in each of the following examples, by middle latitude method:

| | Lat. from and lat. in. | Long. from and long. in. | Answers. Course and dist. |
|---|---|---|---|
| 93. | lat. A 49° 52′ S. | long. A 17° 22′ W. | N. 26° 40′ E. |
| | lat. B 42 13 S. | long. B 11 50 W. | 513·6 |
| 94. | lat. A 21 15 S. | long. A 0 30 W. | S. 14° 37′ E. |
| | lat. B 30 27 S. | long. B 2 10 E. | 570·5 |
| 95. | lat. A 60 15 S. | long. A 14 55 E. | S. 32° 50′ E. |
| | lat. B 65 36 S. | long. B 22 30 E. | 382 |

*Rule IV.* (*Middle Latitude method*).

To find the latitude and longitude in, having given the course from a given place, and distance.

(1.) To find T. D. lat. In fig. p. 151, given the course A and distance AF, to find AO, the T. D. lat.

T. D. lat. = dist. . cos. course.

Apply T. D. lat. to lat. from, and thus get lat. in, then find middle lat.

(2.) To find diff. long.

In same fig., dep. = dist. . sin. course

and in fig. p. 29, $\frac{TS}{UZ} = \cos. TU$

or dep. (nearly) = diff. long. . cos. mid. lat.

hence, by substituting the value of departure,

diff. long. . cos. mid. lat. = dist. . sin. course

$\therefore$ diff. long. = dist. . sin. course . sec. mid. lat.

Apply diff. long. to long. from, and thus get long. in.

*Rule IV.* (*in words*).

(1.) To find latitude in. Add together log. cos. course* and log. distance; the sum (rejecting 10 in the index) is the log. true difference latitude, which find from tables, and mark N. or S. according as the course is northward or southward.

Apply true difference latitude (turned into degrees and minutes, if necessary) to the latitude from, and thus get latitude in. (Rule *e*.) Find the middle latitude. (Rule *c*.)

2. To find longitude in. Add together log. sin.

* Take out at the same time log. sin. course.

course, log. distance, and log. secant middle latitude; the sum (rejecting 20 in the index) is the log. difference longitude, which find in tables, and mark E. or W. according as the course is eastward or westward. Apply the difference longitude (in degrees and minutes) to the longitude from, and thus get longitude in. (Rule *f*.)

EXAMPLE.

96. Sailed from A, S. 37° 10′ W., 472·6 miles; required the latitude in and longitude in (by middle lat. method).

lat. A 27° 20′ S. long. A 25° 12′ W.

(1.) T. D. lat. = dist. . cos. course.

(2.) diff. long. = dist. . sin. course . sec. mid. lat.

log. cos. course 9·901394
log. dist. ..... 2·674494
log. T. D. lat. . 2·575888
∴ T. D. lat. 376·6′
or 6° 17′ S.
lat. from 27 20 S.
lat. in . . 33 37 S.
2)60 57
mid. lat. 30 28

log. sin. course . 9·781134
log. dist. ..... 2·674494
log. sec. mid. lat. 0·064531
log. diff. long. . . 2·520159
∴ diff. long. 331·3′
or 5° 31′ W.
long. from 25 12 W.
long. in. . . 30 43 W.

Required the latitude and longitude in, by middle latitude method, in each of the following examples, having sailed from A as follows:

| | Course and dist. from A. | Lat. A. | Long. A. | Answers. Lat. in. | Long. in. |
|---|---|---|---|---|---|
| 97. | N. 25° 42′ W. 427·3′ | 64° 10′ N. | 40° 15′ W. | 70° 35′ N. | 48° 17′ W. |
| 98. | S. 48 58 W. 175·2 | 47 10 N. | 32 15 W. | 45 15 N. | 35 26 W. |
| 99. | N. 34 48 W. 383·7 | 50 25 N. | 3 40 E. | 55 40 N. | 2 24 W. |

### *Parallel Sailing.*

In parallel sailing the ship is supposed to be kept on a parallel of latitude, as TS, fig. p. 29. The course will evidently be due east or due west, and the distance between two places, as T and S, will be the arc TS between the two meridians passing through the places.

Problems in parallel sailing are solved by means of formula [4], p. 122, namely, dist. = diff. long.. cos. lat., any two terms of which being given, the third may be found.

### *Rule V. (Parallel Sailing).*

To find the course and distance, having given the latitude of the two places, and their longitudes.

(1.) Find the difference longitude.

(2.) The course is evidently due east or due west.

(3.) To find distance.

In fig. p. 29, $\frac{TS}{UZ} = \text{cos. TU}$

or dist. = diff. long.. cos. lat.

### *Rule V. (in words).*

(1.) Find the difference longitude.

(2.) The course is evidently due east or due west, according as the longitude in is to the east or west of longitude from.

(3.) To find the distance. Add together log. cos. latitude and log. difference longitude; the sum (rejecting 10 in index) is the log. distance, which find in the table.

EXAMPLES.

100. Required the course and distance from A to B.

| | |
|---|---|
| lat. A 80° N. | long. A 3° 50′ E. |
| lat. B 80 N. | long. B 6 10 W. |

| | |
|---|---|
| long. from .... 3° 50′ E. | dist. = diff. long. . . cos. lat. |
| long. in ..... 6 10 W. | log. cos. lat. .... 9·239670 |
| 10 0 | log. diff. long. . . 2·778151 |
| 60 | log. dist. .... 2·017821 |
| 600 W. | ∴ dist. 104·2′ |

∴ the course is west.

Required the course and distance from A to B, in each of the following examples:

| | Lat. A and B. | Long. A. | Long. B. | Answers. Course and dist. |
|---|---|---|---|---|
| 101. | 70° 10′ S. | 15° 10′ E. | 22° 15′ E. | East 144·2′ |
| 102. | 50 48 N. | 5 0 W. | 5 0 E. | East 379·2 |
| 103. | 50 10 N. | 40 25 W. | 50 10 W. | West 374·7 |
| 104. | 48 10 N. | 100 0 W. | 110 0 W. | West 400·2 |
| 105. | 75 13 N. | 15 20 E. | 0 0 E. | West 234·7 |
| 106. | 80 15 N. | 179 0 E. | 176 0 W.* | East 50·8 |

### *Rule VI. (Parallel Sailing).*

To find the longitude in, having given the course and distance, and latitude and longitude from.

In fig. p. 29, $\frac{TS}{UZ} = \cos. TU$

$\therefore$ $UZ = TS \,.\, \sec. TU$

or diff. long. = dist. . sec. lat.

Mark difference longitude E. or W. according as the course is E. or W. Apply difference longitude to longitude from, and thus get longitude in.

* In this example it is evident we must modify the general rule; for the diff. long. is never considered to be greater than 180°. When, therefore, the above rule gives the diff. long. greater than 180°, subtract it from 360°, and apply thereto a contrary letter to the one directed by the rule; the result will be the diff. long. to be used.

*Rule VI.* (*in words*).

Add together log. sec. lat. and log. distance, the sum (rejecting 10 in the index) will be the log. difference longitude. Find the natural number thereof, and turn it into degrees, and mark it E. or W. according as the course is E. or W. Apply difference longitude to longitude from, and thus find longitude in. (Rule *f*.)

The latitude in is the same as the latitude from.

EXAMPLE.

107. Sailed from A due east 1000 miles, required the latitude and longitude in.

lat. A 32° 10′ S. long. A 28° 42′ W.

diff. long. = dist. . sec. lat.

log. sec. lat. ..... 0·072372

log. dist. ..... 3·000000

log. diff. long. .... 3·072372

∴ diff. long. 1181′, or 19° 41′ E.

long. from ..... 28 42 W.

∴ long. in. ..... 9 1 W.

and latitude in = lat. from = 32° 10′ S.

Required the latitude and longitude in, in each of the following examples:

| | Course and dist. | Lat. from. | Long. from. | Answers. Lat. in. | Long. in. |
|---|---|---|---|---|---|
| 108. | East 492·5′ | 52° 10′ N. | 0° 29 W. | 52° 10′ N. | 12° 54 E. |
| 109. | East 1752 | 60 0 N. | 5 10 W. | 60 0 N. | 53 14 E. |
| 110. | East 560 | 57 32 N. | 13 5 W. | 57 32 N. | 4 18 . |
| 111. | West 740 | 60 0 N. | 50 0 W. | 60 0 N. | 74 40 W. |

The preceding rules are the principal ones used in navigation. It would be easy for the mathematical student to make for himself others, by means of the relations between the several terms course, dist., dep., &c. as shewn by the fig. p. 151. Thus also in the expression for parallel sailing, namely, dist. = diff. long. . cos. lat. (p. 122), any two of these terms being given the third may be found; as in the following examples:

Sailed from A, in long. 3° 10′ W., 300 miles due east, and altered my longitude 10 degrees; required the latitude and longitude in.

To find latitude.

$$\text{cos. lat.} = \frac{\text{dist.}}{\text{diff. long.}} = \frac{300}{600} = \tfrac{1}{2}$$

$\therefore$ lat. in = 60°, and long. in = 6° 50′ E.

Wishing to make a small island, I took the ship to windward of it in the same latitude with the island, namely, 50° 48′ N. The longitude of the ship by chronometer was 20° 35′ W., and the long. of the island was 23° 50′ W. What was my distance from the island?

In this example of parallel sailing we have given lat. 50° 48′, and diff. long. 3° 15′, or 195′, to find distance.

dist. = diff. long. . cos. lat.

| | |
|---|---|
| log. diff. long. ..... | 2·290035 |
| log. cos. lat. ..... | 9·800737 |
| log. dist. ..... | 2·090772 |

$\therefore$ dist. 123·2 miles.

### *The Day's Work.*

To find the place of the ship at noon, that is, its latitude and longitude, having given the latitude and longitude at the preceding noon, the compass courses, and distances run in the interval, the deviation of the compass for each course on account of local attraction, the variation of the compass, the leeway, the velocity and direction of current (if any), &c., constitutes what is called the Day's Work.

### *Rule VII.* (*the Day's Work*).

(1.) Correct each course for variation (Rule *g*), deviation (Rule *i*), and leeway (Rule *l*); thus get the true courses, and arrange the same in a tabular form, as in the example, p. 168. Add together the hourly distances sailed on each course, and insert the same in table opposite the true course.

(2.) Take out of the traverse table the true difference latitude and departure for each course and distance, putting them down in the columns headed with the same letters as in course. Previously to opening the traverse table, fill up the columns of true difference latitude and departure not wanted, by drawing horizontal lines; this will frequently prevent mistakes.

(3.) If the ship does not sail from a place whose latitude and longitude are known, her bearing and distance from some near object, as a church-spire, &c. must be ascertained, and also its latitude and longitude. Then the ship is supposed to sail from this known object to her anchorage, her course being the opposite to the bearing of the object from the ship. This course must be corrected like the rest for variation and deviation, and

inserted in the table as an actual course, with the distance of the object as a distance.

(4.) If a current sets the ship in any ascertained direction, and with a known velocity, these also may be conceived to be an independent course and distance, and must be corrected for variation, and should be for deviation also, if the latter correction is appreciable, which is rarely the case.

(5.) To find the latitude in. The quantities in the four columns of true difference latitude and departure being added up separately, the difference between the north difference of latitude and south difference of latitude, with the name of the greater, will give the true difference of latitude made at the end of the day. The departure is found in a similar manner. Apply true difference latitude to latitude from, so as to obtain the latitude in.

(6.) To find the longitude in (since diff. long.= dep. . sec. mid. lat. [125]), add together log. sec. mid. lat. and log. departure, the result (rejecting 10 in the index) is the log. difference longitude. Find this in table, and thus the longitude in is found.*

The following example, worked out in detail, will perhaps be sufficient to explain the operations directed in the above general rule.

* Or thus: (without using mid. lat.)

$$\text{since } \frac{\text{diff. long.}}{\text{M. D. lat.}} = \frac{\text{dep.}}{\text{T. D. lat.}} \quad \text{(fig. p. 151.)}$$

$$\therefore \text{ diff. long.} = \frac{\text{M. D. lat.} \,.\, \text{dep.}}{\text{T. D. lat.}}$$

To find diff. long., add together log. M. D. lat. and log. dep., and from the sum subtract log. T. D. lat.; the remainder is the log. diff. long., which find in the tables.

EXAMPLE.

112. April 27th, 1849, at noon. A point of land in latitude 36° 30′ S. and longitude 110° 20′ W. bore by compass E.N.E.$\frac{1}{2}$E. (ship's head being S.E. by S. by compass), distant 14 miles; afterwards sailed as by the following log. account; required the latitude and longitude in, on April 28th, at noon.

| Hours. | Knots. | Courses. | Winds. | Lee-way. | Devi-ation. | Remarks. |
|---|---|---|---|---|---|---|
| 1 | 2·5 | S.W.$\frac{1}{2}$W. | S.b.E. | 2$\frac{1}{4}$ | $\frac{1}{2}$ l. | P.M. |
| 2 | 3·4 | | | | | |
| 3 | 2·3 | | | | | |
| 4 | 3·2 | W.b.S.$\frac{1}{2}$S. | S.b.W. | 2$\frac{1}{2}$ | $\frac{3}{4}$ l. | |
| 5 | 4·4 | | | | | |
| 6 | 2·3 | | | | | Variation of |
| 7 | 2·3 | W.b.N.$\frac{3}{4}$N. | S.W. | 2 | $\frac{3}{4}$ l. | compass 1$\frac{3}{4}$ E. |
| 8 | 3·3 | | | | | |
| 9 | 4·0 | | | | | |
| 10 | 5·4 | | | | | |
| 11 | 4·2 | | | | | |
| 12 | 4·4 | | | | | |
| 1 | 3·3 | N.W.$\frac{1}{2}$W. | W.b.S.$\frac{3}{4}$S. | 2$\frac{1}{2}$ | $\frac{1}{2}$ l. | A.M. |
| 2 | 3·3 | | | | | |
| 3 | 3·5 | | | | | |
| 4 | 4·2 | | | | | A current set the |
| 5 | 6·3 | W.b.S. | S.$\frac{1}{2}$W. | 1$\frac{1}{2}$ | $\frac{3}{4}$ l. | ship the last 8 |
| 6 | 3·7 | | | | | hours, by com- |
| 7 | 2·5 | | | | | pass E.$\frac{1}{2}$S., 2 |
| 8 | 5·0 | | | | | miles an hour. |
| 9 | 5·2 | S.W. | S.b.E. | 2$\frac{1}{2}$ | $\frac{1}{2}$ l. | |
| 10 | 3·4 | | | | | |
| 11 | 6·3 | | | | | |
| 12 | 5·4 | | | | | |

(1.) The column in the above table headed deviation should be formed from the general table of deviations (p. 145) previously to correcting courses. Thus, in the first course in the preceding table, the ship's head is

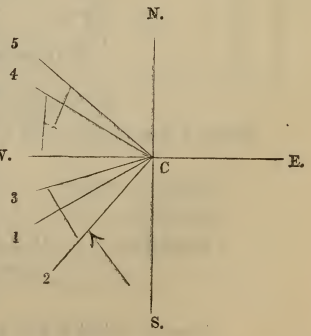
S.W.$\frac{1}{2}$W.; looking in the deviation table we see that the corresponding correction is $\frac{1}{2}$ W. or $\frac{1}{2}$ l. (Rule *i.*)

(2.) Form a table such as below, by writing down the headings, points, courses, &c. over the seven columns which are to be filled in with the corrected courses, &c.

| Points. | Courses. | Distance. | Diff. lat. | | Departure. | |
|---|---|---|---|---|---|---|
| | | | N. | S. | E. | W. |
| | | | | | | |

(3.) To correct the courses.

The courses are more readily corrected by drawing two lines at right angles, to represent the N., S., E., and W. points of the compass, and then a line to represent (roughly) the compass course of the ship. The direction in which the correction for leeway must be applied will then be easily seen.




After some experience in correcting courses, they can be made mentally, and the diagram dispensed with.

To correct the departure course, which is W.S.W.$\frac{1}{2}$W. (the opposite bearing to E.N.E.$\frac{1}{2}$E.).

Draw a line roughly in the fig. W.S.W.$\frac{1}{2}$W. as C1, it is then seen that

| | pts. | qrs. | |
|---|---|---|---|
| compass course..... | 6 | 2 | r. S. |
| variation ..... | 1 | 3 | r. |
| ship's head S.E.b.S..dev. | 0 | 0 | |
| | 1 | 3 | r. |
| true course..... | 8 | 1 | r. S. |

(See table, p. 145.)

or 7 pts. 3 qrs. left of N., or W.$\frac{1}{4}$N. dist. 14′.

Insert this course and distance in table below.

| Points. | Courses. | Dist. | Diff. lat. N. | Diff. lat. S. | Departure. E. | Departure. W. |
|---|---|---|---|---|---|---|
| 7¾ | W.$\frac{1}{4}$N. | 14·0 | 0·7 | — | — | 14·0 |
| 8 | W. | 8·2 | — | — | — | 8·2 |
| 6 | W.N.W. | 9·9 | 3·8 | — | — | 9·2 |
| 3¼ | N.W.b.N.$\frac{1}{4}$W. | 23·6 | 19·0 | — | — | 14·1 |
| ¾ | N.$\frac{3}{4}$W. | 14·3 | 14·1 | — | — | 2·1 |
| 6½ | W.N.W.$\frac{1}{2}$W. | 17·5 | 5·1 | — | — | 16·7 |
| 7¾ | W.$\frac{1}{4}$S. | 20·3 | — | 1·0 | — | 20·3 |
| 5¾ | S.E.b.E.$\frac{3}{4}$E. | 16·0 | — | 6·8 | 14·5 | — |
| | | | 42·7 | 7·8 | 14·5 | 84·6 |
| | | | 7·8 | | | 14·5 |
| | | T. D. lat. 34·9 N. | | | Dep. 70·1 W. | |

1st Course S.W.$\frac{1}{2}$W.

Draw a line in fig. S.W.$\frac{1}{2}$W. as C2; then

| | pts. | qrs. | |
|---|---|---|---|
| compass course ..... | 4 | 2 | r. S. |
| variation..... | 1 | 3 | r. |
| deviation..... | 0 | 2 | l. |
| | 1 | 1 | r. |
| | 5 | 3 | r. S. |
| leeway (wind S.b.E.) .. | 2 | 1 | r. |
| true course ..... | 8 | 0 | r. S. or due W. 8·2′. |

The distance 8·2′ is found by adding up the hourly distances until the course is altered, at 4 o'clock. Insert this course and distance in the table.

2d Course W.b.S.$\frac{1}{2}$S.

Draw a line in fig. W.b.S.$\frac{1}{2}$S. as C3.

| | pts. | qrs. | |
|---|---|---|---|
| compass course ..... | 6 | 2 | r. S. |
| variation..... | 1 | 3 | r. |
| deviation ..... | 0 | 3 | l. |
| | 1 | 0 | r. |
| | 7 | 2 | r. S. |
| leeway ..... | 2 | 2 | r. |
| true course ..... | 10 | 0 | r. S. |
| or | 6 | 0 | l. N. |
| | = W.N.W. 9·9′. | | |

Insert this course and distance in the table.

3d Course W.b.N.$\frac{3}{4}$N.

Draw a line W.b.N.$\frac{3}{4}$N. as C4.

| | pts. | qrs. | |
|---|---|---|---|
| compass course ..... | 6 | 1 | l. N. |
| variation ..... | 1 | 3 | r. |
| deviation ..... | 0 | 3 | l. |
| | 1 | 0 | r. |
| | 5 | 1 | l. N. |
| leeway ..... | 2 | 0 | r. |
| true course ..... | 3 | 1 | l. N. |
| or N.W.b.N.$\frac{1}{4}$W. 23·6′. | | | |

Insert this course and distance in table.

4th Course N.W.$\frac{1}{2}$W.

Draw a line N.W.$\frac{1}{2}$W. as C5.

| | pts. qrs. |
|---|---|
| compass course | 4 2 l. N. |
| variation | 1 3 r. |
| deviation | 0 2 l. |
| | 1 1 r. |
| | 3 1 l. N. |
| eeway | 2 2 r. |
| true course | 0 3 1 N. |
| | or N.$\frac{3}{4}$W. 14·3′. |

Insert this course and distance in the table.

Proceed with the 5th and 6th courses in the same manner, thus:

5th Course.

| pts. qrs. | |
|---|---|
| W.b.S. | 7 0 r. S. as C3. |
| | 1 3 r. |
| | 0 3 l. |
| | 1 0 r. |
| | 8 0 r. S. |
| | 1 2 r. |
| | 9 2 r. S. |
| or | 6 2 l. N. |
| = W.N.W.$\frac{1}{2}$W. 17·5′. | |

6th Course.

| pts. qrs. | |
|---|---|
| S.W. | 4 0 r. S. |
| | 1 3 r. |
| | 0 2 l. |
| | 1 1 r. |
| | 5 1 r. S. |
| | 2 2 r. |
| | 7 3 r. S. |
| or W.$\frac{1}{4}$S. 20·3′. | |

| | pts. qrs. |
|---|---|
| Current course E.$\frac{1}{2}$S. | 7 2 l. S. |
| variation | 1 3 r. |
| true course | 5 3 l. S. |
| | or S.E.b.E.$\frac{3}{4}$E. 16·0′. |

Previously to opening the traverse table to take out the difference latitude and departure corresponding to

each course and distance in the above table, fill up th columns not wanted: thus in the first course W.$\frac{1}{4}$N. the N. and W. columns will be wanted; fill up the S. and E. columns by drawing a line under S. and E. In the second course W. the three columns N., S., and E. will not be wanted; fill them up with lines. In the same manner proceed with the other courses.

To find difference latitude and departure for each course and distance, by traverse table.

Enter traverse table, and take out the difference latitude and departure corresponding to $7\frac{3}{4}$ points, and distance 14·0. (Look out rather $7\frac{3}{4}$ points and 140 distance, the diff. lat. and dep. for which are 6·9 and 139·8; move the decimal points one place to the left,) and put down the result to the nearest tenth, which are ·7 and 14·0. Insert them in the spaces left unmarked under N. and W.

The second course being due W. 8·2′, the departure will be 8·2 (the same as the distance).

With third course 6 points and distance 9·9 (looking for 99, and making the proper change in decimal points) the diff. lat. is 3·8′ and dep. 9·2′.

In a similar manner find difference latitude and departure for the other courses.

When the four columns are added up, it appears that the ship has sailed N. 42·7′ and S. 7·8′, therefore upon the whole the true difference latitude is 34·9′ N.; and her departure has been 14·5′ E. and 84·6′ W., hence the departure made good in the 24 hours is 70·1′ W.

To find the latitude in, apply the true difference latitude to the latitude from, in the usual manner, to obtain the latitude in. (Rule *e*.)

To find the longitude in.* With the latitude from and latitude in, find middle latitude. Then,

$$\text{since } \frac{\text{dep.}}{\text{diff. long.}} = \text{cos. mid. lat.} \quad \text{(p. 125)}$$

$$\therefore \text{ diff. long.} = \text{dep.} \,.\, \text{sec. mid. lat.}$$

(Or in words). To find difference longitude, add together log. secant mid. lat. and log. departure; the result (rejecting 10 in index) is the log. difference longitude, which, found in the tables, and applied to the longitude from, gives the longitude in. Thus:

| To find latitude in. | | To find longitude in. | |
|---|---|---|---|
| T. D. lat. | 0° 34′ 54″ N. | log. sec. mid. lat. | 0·093148 |
| lat. from | 36 30 0 S. | log. departure .. | 1·845718 |
| lat. in .. | 35 55 6 S. | log. diff. long... | 1·938866 |
| | 2)72 25 6 | ∴ diff. long. | 87′ |
| mid. lat. . | 36 12 33 | or | 1° 27′ W. |
| | | long. from .. | 110 20 W. |
| | | long. in..... | 111 47 W. |

* Or thus: To find long. in (by inspection).

$$\text{Since } \frac{\text{dep.}}{\text{dist.}} = \text{sin. course}$$

$$\text{and } \frac{\text{dep.}}{\text{diff. long.}} = \text{cos. mid. lat.} = \text{sin. compl. mid. lat.}$$

If, therefore, the traverse table is entered with complement of mid. lat. as a course, and with the given departure, the distance corresponding thereto will be the difference of longitude nearly.

To find course and distance on Mercator's chart between two known places.

To find course. Apply the edge of a ruler to the two places, and then ascertain at what degree a straight line (as the edge of the same or another ruler) placed parallel thereto, and passing through the center of some adjacent compass, cuts the circumference. This will indicate the bearing or course required.

To find distance. This may, in general, be found by applying the distance on the chart to the side of chart, so that the chart distance may be so placed that the middle point may coincide with the middle parallel between the two places: then the degrees of lat. covered by the chart distance will be the distance nearly.

More concise methods for solving many of the preceding problems might have been given, by employing the traverse table, &c. But these more properly belong to works written exclusively on the subject of navigation.

The following examples formed part of the examination papers recently given at the Royal Naval College, at the monthly examination of mates in navigation :—

113. Required the course and distance from A to B.
lat. A... 50° 25′ N. long. A... 3° 40′ E.
lat. B... 55 40 N. long. B... 2 25 W.

114. Find the course (by compass) and dist. from A to B.
lat. A... 70° 10′ N. long. A... 15° 5′ E.
lat. B... 70 10 N. long. B... 20 5 E.

The deviation is given in table, p. 145.

115. June 20, 1848, at noon, a point of land in latitude 47° 12′ N. and longitude by account 3° 10′ W. bore by compass E.N.E., the ship's head being South by compass, distant 17 miles (variation of the compass $3\frac{1}{4}$ W.); afterwards sailed as by the following log account: required the lat. and long. in, on June 21, at noon.

| H. | K. | $\frac{1}{10}$ths. | Course. | Wind. | Lee-way. | |
|---|---|---|---|---|---|---|
| 1 | 3 | 5 | E.b.N.$\frac{1}{4}$N. | N.$\frac{1}{2}$E. | 2 | |
| 2 | 3 | 2 | | | | |
| 3 | 4 | 0 | | | | |
| 4 | 3 | 6 | | | | Variation of the compass $3\frac{1}{4}$W. For local deviation see table, p. 145. |
| 5 | 3 | 5 | W.N.W. | ditto | $1\frac{3}{4}$ | |
| 6 | 4 | 1 | | | | |
| 7 | 3 | 6 | | | | |
| 8 | 3 | 7 | | | | |
| 9 | 4 | 2 | S.S.W. | West | $2\frac{1}{4}$ | |
| 10 | 4 | 1 | | | | |
| 11 | 3 | 6 | | | | |
| 12 | 3 | 2 | | | | |
| | | | | | | |
| 1 | 3 | 6 | N.N.W. | ditto | $1\frac{3}{4}$ | |
| 2 | 3 | 2 | | | | A current set the ship the last 4 hours, by comp. S.S.W.$\frac{1}{2}$W., 2 miles an hour. The current need not be corrected for local deviation. |
| 3 | 4 | 0 | | | | |
| 4 | 4 | 3 | | | | |
| 5 | 4 | 2 | S.E.$\frac{1}{2}$E. | S.S.W. | $1\frac{1}{2}$ | |
| 6 | 3 | 6 | | | | |
| 7 | 3 | 6 | | | | |
| 8 | 4 | 2 | | | | |
| 9 | 3 | 5 | S.W.$\frac{3}{4}$W. | S.b.E. | $3\frac{1}{4}$ | |
| 10 | 4 | 2 | | | | |
| 11 | 4 | 0 | | | | |
| 12 | 5 | 2 | | | | |

116. Required the course and distance from A to B.

lat. A... 47° 50′ N. long. A... 32° 20′ W.

lat. B... 45 10 N. long. B... 35 40 W.

117. Required the course (by comp.) and dist. from A to B.

lat. A... 70° 10′ S. long. A... 5° 0′ W.

lat. B... 70 10 S. long. B... 5 0 E.

Variation of compass $2\frac{1}{2}$ W. For local dev. see p. 145.

118. March 5, 1848, at noon, a point of land in latitude 57° 12′ N. and longitude by account 75° 34′ W. bore by compass E.N.E. (ship's head being N. by compass), distant 18 miles (variation of the compass $1\frac{1}{2}$W.); afterwards sailed as by the following log account: required the lat. and long. in, on March 6, 1848, at noon.

| H. | K. | $\frac{1}{10}$ths. | Course. | Wind. | Lee-way. | |
|---|---|---|---|---|---|---|
| 1 | 5 | 0 | N.$\frac{1}{2}$W. | W.b.N. | $1\frac{3}{4}$ | |
| 2 | 4 | 5 | | | | |
| 3 | 4 | 2 | | | | |
| 4 | 3 | 7 | | | | Variation of the compass $1\frac{1}{2}$W. For local deviation see table, p. 145. |
| 5 | 2 | 0 | S.W.b.S. | ditto | $2\frac{1}{4}$ | |
| 6 | 2 | 3 | | | | |
| 7 | 2 | 4 | | | | |
| 8 | 3 | 0 | N.N.E.$\frac{1}{2}$E. | N.W.$\frac{1}{2}$N. | 2 | |
| 9 | 3 | 2 | | | | |
| 10 | 2 | 5 | | | | |
| 11 | 3 | 2 | | | | |
| 12 | 2 | 6 | | | | |
| | | | | | | |
| 1 | 2 | 0 | W.$\frac{1}{2}$N. | N.N.W. | $2\frac{3}{4}$ | |
| 2 | 2 | 6 | | | | A current set the ship the last 6 hours, 4 miles an hour, N.N.W. (by compass). The current need not be corrected for local deviation. |
| 3 | 2 | 1 | | | | |
| 4 | 2 | 7 | | | | |
| 5 | 5 | 0 | S.E.$\frac{1}{2}$E. | S.W. | 0 | |
| 6 | 6 | 2 | | | | |
| 7 | 6 | 3 | | | | |
| 8 | 7 | 0 | | | | |
| 9 | 1 | 0 | S.$\frac{1}{2}$W. | W.S.W. | 3 | |
| 10 | 1 | 2 | | | | |
| 11 | 1 | 5 | | | | |
| 12 | 1 | 6 | | | | |

119. Required the course and distance from A to B.
lat. A... 72° 20′ S. long. A... 13° 25′ W.
lat. B... 65 42 S. long. B... 20 10 W.

120. Find the course (by compass) and dist. from A to B.
lat. A... 70° 10′ N. long. A... 15° 5′ E.
lat. B... 70 10 N. long. B... 20 5 E.
Variation of compass 2 W. For local deviation see p. 145.

121. June 1, 1848, in longitude 18° 28′ E., and latitude 34° 28′ S., a point of land bore N.W. (ship's head N. by compass), distant 10 miles (variation of the compass $2\frac{1}{4}$ W.); afterwards sailed as per log: required the latitude and longitude in, on June 2, 1848.

| H. | K. | $\frac{1}{10}$ths. | Course. | Wind. | Lee-way. | |
|---|---|---|---|---|---|---|
| 1 | 5 | 4 | N.b.E.$\frac{1}{2}$E. | N.W.$\frac{1}{2}$W. | $2\frac{1}{4}$ | |
| 2 | 5 | 2 | | | | |
| 3 | 5 | 8 | | | | |
| 4 | 6 | 1 | | | | Variation of the compass $2\frac{1}{4}$W. For local deviation see table, p. 145. |
| 5 | 6 | 5 | S.S.W. | N.W. | $\frac{1}{4}$ | |
| 6 | 7 | 3 | | | | |
| 7 | 7 | 0 | | | | |
| 8 | 7 | 2 | | | | |
| 9 | 6 | 8 | N.W.b.W. | S.E. | 0 | |
| 10 | 6 | 5 | | | | |
| 11 | 6 | 1 | | | | |
| 12 | 5 | 8 | S.b.W.$\frac{3}{4}$W. | S.E.$\frac{1}{4}$E. | $2\frac{1}{4}$ | |
| | | | | | | |
| 1 | 6 | 0 | | | | |
| 2 | 6 | 5 | | | | A current set the ship the last 5 hours, by compass N.W., 2 miles an hour. The current need not be corrected for local deviation. |
| 3 | 6 | 8 | | | | |
| 4 | 6 | 4 | N.N.E. | N.W. | 2 | |
| 5 | 6 | 0 | | | | |
| 6 | 6 | 5 | | | | |
| 7 | 6 | 8 | N.W. | East | 0 | |
| 8 | 2 | 4 | | | | |
| 9 | 3 | 6 | | | | |
| 10 | 4 | 7 | | | | |
| 11 | 3 | 5 | | | | |
| 12 | 2 | 2 | | | | |

122. Required the course and distance from A to B.

lat. A... 3° 30′ N. long. A... 74° 40′ E.

lat. B... 2 20 S. long. B... 59 17 E.

123. A ship sailed from A 380 miles E.S.E.$\frac{1}{2}$E. : required the latitude and longitude in.

lat. A... 39° 12′ N. long. A... 78° 50′ W.

124. May 2, 1849, at noon, a point of land, in latitude 55° 10′ S. and longitude 67° 20′ W., bore by compass N.E.$\frac{3}{4}$E., ship's head N., distant 12 miles (variation of the compass $3\frac{3}{4}$W.); afterwards sailed as by the following log account: required the latitude and longitude in, on May 3, 1849.

| H. | K. | $\frac{1}{10}$ths. | Course. | Wind. | Lee-way. | |
|---|---|---|---|---|---|---|
| 1 | 3 | 8 | N. | W.N.W. | $3\frac{1}{2}$ | |
| 2 | 3 | 7 | | | | |
| 3 | 4 | 1 | | | | |
| 4 | 3 | 9 | W.S.W.$\frac{3}{4}$W. | N.W.b.N. | $2\frac{3}{4}$ | Variation of the compass $3\frac{3}{4}$W. For local deviation see table, p. 145. |
| 5 | 3 | 8 | | | | |
| 6 | 4 | 0 | | | | |
| 7 | 4 | 1 | | | | |
| 8 | 3 | 7 | N.W.$\frac{1}{2}$W. | N.N.E. | $2\frac{1}{2}$ | |
| 9 | 2 | 9 | | | | |
| 10 | 2 | 8 | | | | |
| 11 | 2 | 5 | | | | |
| 12 | 1 | 7 | E. | S.b.E.$\frac{1}{2}$E. | $2\frac{1}{2}$ | |
| | | | | | | |
| 1 | 1 | 7 | | | | |
| 2 | 1 | 6 | | | | A current set the ship the last 5 hours, by compass, $4\frac{1}{2}$ miles an hour, E.$\frac{1}{2}$S. The current need not be corrected for local deviation. |
| 3 | 2 | 4 | | | | |
| 4 | 2 | 5 | S.S.W.$\frac{3}{4}$W. | S.E.$\frac{1}{2}$S. | 3 | |
| 5 | 2 | 7 | | | | |
| 6 | 3 | 9 | | | | |
| 7 | 3 | 8 | | | | |
| 8 | 2 | 7 | | | | |
| 9 | 3 | 4 | N.$\frac{3}{4}$E. | E.N.E. | 4 | |
| 10 | 4 | 3 | | | | |
| 11 | 4 | 3 | | | | |
| 12 | 4 | 6 | | | | |

125. Required the course and distance from A to B.
lat. A... 21° 15′ S. long. A... 0° 30′ W.
lat. B... 30 27 S. long. B... 2 10 E.

126. Required the compass course and dist. from A to B.
lat. A... 15° 30′ N. long. A... 2° 10′ E.
lat. B... 15 30 N. long. B... 3 40 E.

Var. of compass $2\frac{1}{2}$ points E. For local dev. see p. 145.

127. March 6, 1849, at noon, a point of land, in latitude 47° 10′ N. and longitude by account 10° 46′ W., bore by compass E.b.N.$\frac{1}{2}$N. (ship's head being S.S.E. by compass), distant 20 miles (variation of the compass $2\frac{1}{4}$ W.); afterwards sailed as by the following log account: required the lat. and long. in, on March 7, at noon.

| H. | K. | $\frac{1}{10}$ths. | Course. | Wind. | Lee-way. | |
|---|---|---|---|---|---|---|
| 1 | 3 | 2 | S.S.W.$\frac{1}{2}$W. | S.E. | 2 | |
| 2 | 3 | 5 | | | | |
| 3 | 4 | 0 | | | | |
| 4 | 4 | 0 | S.W. | S.E.b.S. | $1\frac{1}{4}$ | Variation of the compass $2\frac{1}{4}$W. For local deviation see table, p. 145. |
| 5 | 3 | 6 | | | | |
| 6 | 3 | 7 | | | | |
| 7 | 3 | 2 | E.N.E. | S.E. | $2\frac{1}{2}$ | |
| 8 | 3 | 0 | | | | |
| 9 | 4 | 0 | | | | |
| 10 | 4 | 2 | | | | |
| 11 | 7 | 0 | N.E. | S.E. | 0 | |
| 12 | 8 | 2 | | | | |
| | | | | | | |
| 1 | 7 | 2 | N.E.$\frac{1}{4}$E. | N.N.W. | 2 | |
| 2 | 3 | 4 | | | | A current set the ship the last 4 hours, by compass W.b.N., 2 miles an hour. The current need not be corrected for local deviation. |
| 3 | 4 | 2 | | | | |
| 4 | 3 | 7 | | | | |
| 5 | 4 | 0 | N.E.b.E. | N.b.W. | $2\frac{1}{4}$ | |
| 6 | 3 | 6 | | | | |
| 7 | 3 | 7 | | | | |
| 8 | 4 | 2 | | | | |
| 9 | 4 | 0 | | | | |
| 10 | 8 | 0 | N.W. | South. | 0 | |
| 11 | 9 | – | | | | |
| 12 | 10 | – | | | | |

128. Required the course and distance from A to B.

lat. A... 9° 30′ S. long. A... 2° 4′ W.

lat. B... 7 10 S. long. B... 1 30 E.

129. Two places in the same latitude N., whose difference of longitude is 700 miles, are distant from each other 400 miles; required the latitude they are in.

130. Jan. 10, 1848, at noon, a point of land, in latitude 46° 12′ S. and longitude by account 2° 10′ W., bore by compass E.b.S.$\frac{1}{2}$S., distant 20 miles, the ship's head being East by compass; afterwards sailed as by the following log account: required the latitude and longitude in, on Jan. 11, 1848, at noon.

| H. | K. | $\frac{1}{10}$ths. | Course. | Wind. | Leeway. | |
|---|---|---|---|---|---|---|
| 1 | 3 | 5 | S.W.$\frac{1}{2}$W. | S.b.E.$\frac{1}{2}$E. | 2 0 | |
| 2 | 3 | 2 | | | | |
| 3 | 3 | 6 | | | | |
| 4 | 3 | 7 | | | | Variation of the |
| 5 | 3 | 0 | N.$\frac{3}{4}$E. | E.N.E. | $\frac{1}{4}$ | compass 1$\frac{1}{4}$ E. |
| 6 | 3 | 2 | | | | For local deviation see table, |
| 7 | 3 | 6 | | | | p. 145. |
| 8 | 4 | – | | | | |
| 9 | 3 | 2 | S.b.E.$\frac{1}{2}$E. | S.W.$\frac{1}{2}$W. | 2$\frac{3}{4}$ | |
| 10 | 2 | 5 | | | | |
| 11 | 2 | 1 | | | | |
| 12 | 1 | 6 | | | | |
| | | | | | | |
| 1 | 3 | 2 | W.b.S. | S.b.W. | 2$\frac{1}{4}$ | |
| 2 | 3 | – | | | | A current set the |
| 3 | 2 | 6 | | | | ship the last 5 |
| 4 | 3 | 5 | | | | hours, by com- |
| 5 | 3 | 2 | E.N.E. | S.E. | 2$\frac{1}{2}$ | pass S.W.$\frac{1}{4}$W., |
| 6 | 3 | 5 | | | | 4$\frac{1}{2}$ miles an |
| 7 | 4 | 0 | | | | hour. |
| 8 | 3 | 6 | S.S.W.$\frac{1}{2}$W. | S.E. | 1$\frac{3}{4}$ | The current need |
| 9 | 3 | 7 | | | | not be corrected |
| 10 | 2 | 2 | | | | for local devia- |
| 11 | 2 | 6 | | | | tion. |
| 12 | 2 | 7 | | | | |

In this examination paper the ship's compasses are not supposed to be affected by the iron on board.

131. Required the course and distance from A to B.

lat. A... 78° 12′ N. long. A... 80° 15′ W.

lat. B... 73 50 N. long. B... 89 10 W.

132. Sailed from A 555·6 miles due N.; required the latitude and longitude in.

lat. A... 3° 15′ S. long. A... 100° W.

133. On June 5, 1849, at noon, a point of land, in latitude 70° 15′ N. and longitude 7° 13′ E., bore by compass N.W.$\frac{1}{4}$W., distant 18 miles (variation 1$\frac{1}{2}$W.); afterwards sailed as by the following log account: required the latitude and longitude on June 6 at noon.

| H. | K. | $\frac{1}{10}$ths. | Course. | Wind. | Lee-way. | |
|---|---|---|---|---|---|---|
| 1 | 3 | 5 | N.$\frac{1}{4}$W. | W.b.N.$\frac{3}{4}$N. | 2$\frac{1}{4}$ | P.M. |
| 2 | 3 | 2 | | | | |
| 3 | 2 | 7 | | | | |
| 4 | 3 | – | | | | |
| 5 | 4 | – | N.N.E. | East | 2$\frac{1}{2}$ | |
| 6 | 2 | 5 | | | | |
| 7 | 3 | – | | | | Variation 1$\frac{1}{2}$W. |
| 8 | 2 | 6 | | | | |
| 9 | 3 | – | S.S.W.$\frac{1}{2}$W. | S.E. | 1$\frac{1}{2}$ | |
| 10 | 2 | 5 | | | | |
| 11 | 3 | – | | | | |
| 12 | 2 | – | | | | |
| | | | | | | |
| 1 | 8 | – | S.$\frac{1}{2}$W. | North | 0 | A.M. |
| 2 | 8 | – | | | | |
| 3 | 8 | – | | | | |
| 4 | 8 | – | | | | |
| 5 | 8 | – | | | | |
| 6 | 8 | – | | | | |
| 7 | 8 | – | | | | |
| 8 | 7 | 5 | | | | |
| 9 | 8 | – | | | | |
| 10 | 7 | – | | | | |
| 11 | 8 | – | | | | |
| 12 | 9 | – | | | | |

ANSWERS TO QUESTIONS 113–133.

113. Course N. 34° 49′ W., distance 384·5.
114. Compass course E.$\frac{3}{4}$N., distance 101·8.
115. True courses, &c., S.S.W. 17·0′; E.N.E.$\frac{1}{4}$E. 14·3′; S.W.$\frac{1}{2}$W. 14·9′; S.E. 15·1′; N.W.$\frac{1}{2}$N. 15·1′; E.b.N. 15·6′; S.W.$\frac{1}{4}$W. 16·9′; S.$\frac{3}{4}$E. 8·0′. Lat. in 46° 37′ N., long. in 3° 9′ W.
116. Course S. 40° 42′ W., dist. 211·0.
117. Compass course S.E.b.E.$\frac{3}{4}$E., dist. 203·6.
118. True courses, &c., S.W.$\frac{3}{4}$W. 18·0′; North 17·4′; S.b.E.$\frac{1}{4}$E. 6·7′; N.E.$\frac{1}{4}$N. 14·5′; S.W.$\frac{1}{2}$S. 9·4′; E.S.E.$\frac{1}{4}$S. 24·5′; S.b.E.$\frac{1}{4}$E. 5·3′; N.W.$\frac{1}{2}$W. 24·0′. Lat. in 57° 20′ N., long. in 75° 31′ W.
119. Course N. 19° 52′ W., distance 423·2.
120. Compass course E.S.E.$\frac{1}{2}$E., distance 101·8.
121. True courses, &c., E.S.E. 10·0′; N.N.E. 22·5′; S.b.E. 28·0′; W.$\frac{1}{4}$N. 19·4′; S.b.W.$\frac{1}{4}$W. 25·1′; N. N. E. $\frac{1}{2}$E. 18·9′; W. N. W. $\frac{3}{4}$W. 23·2′; W.N.W.$\frac{1}{4}$W. 10·0′. Lat. in 34° 36′ S., long. in 17° 57′ E.
122. S. 69° 14′ W., 987·1.
123. Lat. in 37° 22′ N., long. in 71° 8′ W.
124. True courses, &c., S.b.W.$\frac{1}{4}$W. 12·0′; North 11·6′; S.$\frac{1}{2}$E. 15·8′; S.W.$\frac{3}{4}$W. 11·9′; N.N.E.$\frac{1}{2}$E. 7·4′; S.b.W.$\frac{1}{2}$W. 15·6′; W.N.W.$\frac{1}{2}$W. 16·6′; N.E.$\frac{3}{4}$E. 22·5′. Lat. in 55° 23′ S., long. in 67° 37′ W.
125. S. 14° 36′ E., 570·4.
126. N.E.$\frac{1}{2}$E. 86·7.
127. True courses, &c., S.W.$\frac{1}{4}$W. 20·0′; S.b.W.$\frac{3}{4}$W. 10·7′; S.S.W.$\frac{1}{2}$W. 11·3′; N.N.E.$\frac{1}{4}$E. 14·4′; N.N.E.$\frac{1}{2}$E. 15·2′; N.E.$\frac{3}{4}$E. 18·5′; E.N.E. 19·5′;

W.N.W.$\frac{3}{4}$W. 27·0′; W.S.W.$\frac{3}{4}$W. 8·0′. Lat. in 47° 26′ N., long. in 11° 3′ W.

128. N. 56° 32′ E., 253·9.

129. Lat. 55° 9′ N. (See similar Ex. p. 163.)

130. True courses, &c., N.W.$\frac{1}{2}$W. 20·0′; W.b.S.$\frac{1}{4}$S. 14·0′; N.N.E.$\frac{1}{4}$E. 13·8′; S.E.b.S.$\frac{1}{4}$S. 9·4′; W.N.W.$\frac{1}{4}$W. 12·3′; E.N.E.$\frac{1}{4}$N. 10·7′; S.W.b.W. 14·8′; S.W.b.W.$\frac{1}{2}$W. 22·5′. Lat. in 46° 6′ S., long. in 3° 24′ W.

131. S. 26° 4′ W., 291·7.

132. Lat. 6° 0′ 36″ N.; long. 100° W.

133. True courses, &c., S.E.b.E.$\frac{3}{4}$E. 18′ dep. course; N.$\frac{1}{2}$E. 12·4′; N.N.W. 12·1′; S.S.W.$\frac{1}{2}$W. 10·5′; S.b.E. 95·5′. Lat. in 68° 48′ N.; long. in 8° 29′ E.

# SOLUTIONS OF PROBLEMS.

## SECTION II.

### PROBLEMS IN ASTRONOMY, &c.

PROB. 102 (Fig. 73.)

LET NWSE represent the horizon, NZS the celestial meridian, WZE the prime vertical (see definitions, p. 23): take NP = 50° 48′, the latitude of spectator; then P is the pole of the heavens, since the altitude of the pole=lat. of place (p. 27), and PZ is the colatitude (p. 26); take PQ= 90°, then Q is a point in the celestial equator: through E and W, the east and west points, and Q draw the circle WQE; this will represent the celestial equator (p. 26). Let X represent the place of the sun at the time of observation, and through X draw XZ a circle of altitude, and XP a circle of declination: then in the spherical triangle PXZ are given, the polar distance PX (= 90° − decl.), the zenith distance XZ (= 90° − altitude), and the colatitude of spectator PZ (= 90°−lat.), to find the angle PZX, the azimuth of the sun (p. 28).

*Calculation by Rule VIII.*

| | | |
|---|---|---|
| PZ .. | 39° 12′ 0″ | 0·199263 |
| ZX .. | 43 40 0 | 0·160860 |
| | 4 28 0 | 4·763976 |
| PX .. | 66 32 15 | 4·712286 |
| | 71 0 15 | 9·836385 |
| | 62 4 15 | ∴ 111° 51′ = PZX |

Hence the azimuth is N. 111° 51′ W., the angle being measured from the north towards the west.

### Prob. 103 (Fig. 73.)

The same construction being made as in the preceding problem, there are given the three sides of the triangle PZX, to find the hour-angle of the sun ZPX, which is also the apparent time, since the heavenly body is west of meridian (p. 29).

*Calculation by Rule VIII.*

| | | |
|---|---|---|
| PX .. | 66° 32′ 15″ | 0·037479 |
| PZ .. | 39 12 0 | 0·199263 |
| | 27 20 15 | 4·763976 |
| ZX .. | 43 40 0 | 4·152340 |
| | 71 0 15 | 9·153058 |
| | 16 19 45 | ∴ 2h. 57m. 16s. = ZPX |

### Prob. 104 (Fig. 73.)

In the fig. are given the polar dist. PX, the zenith distance ZX, and the azimuth PZX, to find the hour-angle P; this is easily done by Rule XII. Or thus:

$$\frac{\text{sin. P}}{\text{sin. az.}} = \frac{\text{sin. zen. dist.}}{\text{sin. pol. dist.}} = \frac{\text{cos. alt.}}{\text{cos. decl.}}$$

∴ sin. P = sin. az. . cos. alt. . sec. decl.

| | | |
|---|---|---|
| log. sin. | 111° 51′ 0″ | 9·967623 |
| ,, cos. | 46 20 0 | 9·839140 |
| ,, sec. | 23 27 45 | 0·037479 |
| | | 9·844242 |

∴ 2h. 57m. 16s. = P

PROB. 105 (Fig. 73.)

In the fig. are given, the polar distance PX = 74° 0′, colatitude PZ = 39° 12′, hour-angle P = 38° 0′; to find the zenith distance ZX, and thence the altitude.

Calculation by Rule X or XI.

PROB. 106.

Construct a similar figure to the one in the preceding problem, but place the heavenly body on the east side of the meridian, since the hour-angle is A.M.

In the fig. are given, the polar dist. PX = 77° 31′, colatitude PZ = 39° 12′, hour-angle P = 2h. 53m. 1s.; to find the angle PZX, the azimuth.

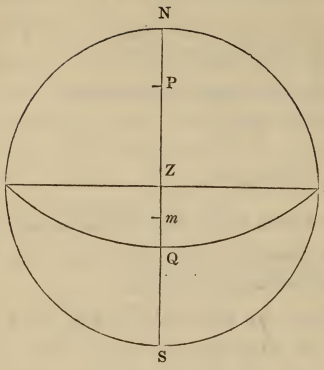
*Calculation by Rule XIII.*

| | | |
|---|---|---|
| 77° 31′ 0″ | 0·401830 | 0·401830 |
| 39 12 0 | 0·280167 | 0·069894 |
| 116 43 0 | 9·975255 | 9·516112 |
| 38 19 0 | 10·657252 | 9·987836 |
| 58 21 30 | 77° 35′ | 44° 12′ |
| 19 9 30 | 44 12 | |

∴ 121 47 = PZX = azimuth.

In the following twelve problems the heavenly body is supposed to be on the meridian at the time of observation: in this position the triangle PZX (fig. 73) vanishes, and the calculations are simplified.

PROB. 107.




Describe the horizon NWSE, the meridian NZS, and the prime vertical WZE: take $Sm = 70°$, to represent the sun's meridian altitude; then $m$ is the place of the sun. (The altitude is measured from the south point of the horizon S, and not from the north, since by the problem the zenith Z of the spectator is *north* of the sun.) And since the declination of the sun is 20° north, the equator must be drawn 20°, to the south of the heavenly body: take therefore $mQ = 20°$, and draw the circle WQE, to represent the celestial equator. Then the latitude of spectator ZQ is found as follows:

$$\text{latitude} = ZQ = Zm + mQ = \text{zen. dist.} + \text{decl.}$$
$$= 90° - 70° + 20° = 40°$$

On the meridian take a point P, 90° distant from the equator, then P (the elevated pole) is the north pole of the heavens; and since Z, the zenith of the spectator, is by the figure north of the equator, the place is in north latitude, $\therefore$ lat. $= 40°$ N.

PROB. 108 (Fig. 66.)

Describe the horizon NWSE, the meridian NZS, and prime vertical WZE. Take a distance $Sm$ on the meridian ($= 70°$) to represent the sun's meridian alti-

tude: then $m$ is the place of the sun. (The altitude is measured from S, the south point of the horizon, since the zenith of spectator is north.) And since the declination is *south* 5°, the equator must be drawn 5° north of the heavenly body: take, therefore, $m\mathrm{Q} = 5°$, and through Q draw WQE, the celestial equator.

$$\text{Lat.} = \mathrm{ZQ} = \mathrm{Z}m - m\mathrm{Q} = \text{zen. dist.} - \text{decl.}$$
$$= 90° - \text{alt.} - \text{decl.} = 90° - 70° - 5° = 15°$$

The latitude is north, since the zenith is north of equator.

Note. In the diagrams fig. 66, &c. the distances are taken roughly; it being sufficient to indicate the form of the figure, and not the correct values of the quantities.

### Prob. 109 (Fig. 67.)

Describe as before the horizon, celestial meridian, and prime vertical. Take a distance N$m$ on the meridian $= 70°$, to represent the star's meridian altitude; then $m$ is the place of the star. (The altitude in this case is measured from N, the north point of the horizon, since the zenith of the spectator is by the problem south of the body.) And since the declination is 25° north, take $m\mathrm{Q} = 25°$, measured from $m$ southward, and draw WQE, the celestial equator.

$$\text{Lat.} = \mathrm{ZQ} = m\mathrm{Q} - m\mathrm{Z} = \text{star's decl.} - \text{zen. dist.}$$
$$= 25° - 20 = 5°$$

The latitude is north, since by the fig. the zenith is north of the equator.

### Prob. 110 (Fig. 68.)

Describe horizon, celestial meridian, and prime vertical. Take N$m$ = 30°, the sun's meridian altitude

(measured from N, since the zenith is south): take $mQ = 10°$, the sun's decl., then a circle through WQE will represent the celestial equator.

$$\text{Lat.} = ZQ = Zm - mQ = \text{zen. dist.} - \text{decl.}$$
$$= 60° - 10° = 50°.$$

The latitude is south, since the zenith is, by the figure, south of the equator, and therefore the spectator is in the southern hemisphere.

PROB. 111 (Fig. to Prob. 107.)

Describe the horizon, celestial meridian, and prime vertical. Since the latitude is 40° N. take $ZQ = 40°$, and draw WQE to represent the celestial equator: from Q measure $Qm = 20°$, then $m$ is the place of the heavenly body, since its declination is 20° N., and

$$mZ = \text{zen. dist.} = ZQ - Qm = 40° - 20° = 20°$$
$$\therefore \quad \text{altitude } Sm = 70°.$$

PROB. 112 (Fig. 68.)

Describe horizon, celestial meridian, and prime vertical. Take $Nm = 30°$, then $m$ is the place of the sun (the altitude being measured from the north, since the zenith is south of the heavenly body): take $ZQ = 50°$ = the latitude of place, and through Q draw WQE to represent the celestial equator (ZQ is taken to the north of Z, since the place is in the southern hemisphere): then $mQ$ is the declination of body; and

$$mQ = mZ - ZQ = \text{zen. dist.} - \text{lat.} = 60° - 50° = 10°.$$

The declination is 10° north, since by the figure the heavenly body is north of the equator.

PROB. 113.

Describe the following parts of fig. 66, the hori-

zon NWSE, celestial meridian NZS, and prime vertical WZE: then since the place is on the equator, the celestial equator must pass through the zenith, and therefore WZE represents also the celestial equator: take a distance $Sm = 57°$, to represent the star's meridian altitude (measured from S, since the zenith must be north of the star); then

$mZ = 33° =$ zenith dist. = declination,

since the equator passes through Z.

The declination is south, $m$ being south of Z.

PROB. 114 (Fig. 69).

Let $m$ be the place of the heavenly body below the pole P, and $m_1$ its place above pole.

Then $Nm =$ star's meridian altitude at inferior transit.

$Nm_1 =$ . . . . superior transit.

and $mP = m_1P =$ star's polar distance.

Now lat. $= NP = Nm + mP$

lat. $= NP = Nm_1 - m_1P$

$\therefore$ 2 lat. $= Nm + Nm_1$ (since $mP = m_1P$)

$\therefore$ lat. $= \frac{1}{2}(Nm + Nm_1) = 45°$ N.

PROB. 115 (Fig. 70).

Describe horizon and celestial meridian. Take $Nm = 20° =$ star's meridian altitude at inferior transit; and since it passes through the zenith at superior transit, bisect $Zm$ in P; then P is the pole, and (as in last proof)

lat. $PN = \frac{1}{2}(Nm + NZ) = \frac{1}{2}(20° + 90°) = 55°$.

PROB. 116.

Construct the horizon NWSE, and celestial meridian NZS. From N take a meridian altitude $= x$, and from Z a meridian zenith distance $= x$;

$\therefore$ meridian altitude at superior transit $= 90° - x$

$\therefore$ lat. $= \frac{1}{2}$(alt. below pole + alt. above pole)
$= \frac{1}{2}(x + 90° - x) = 45°$.

## Prob. 117.

Describe the horizon NWSE, and celestial meridian NZS; on the meridian, take NP = 60°, the latitude; then P is the pole; and since the declination is 40°, the polar distance = 50°; take therefore from P towards N a distance $Pm = 50°$, and from P towards the zenith a distance $Pm_1 = 50°$; then $m$ is the place of the body below the pole, and $m_1$ is the place of body above the pole.

Alt. below pole $= Nm = NP - Pm = 60° - 50° = 10°$
alt. above pole $= Sm_1 = 90° - Zm_1 = 90° - (Pm_1 - PZ)$
$= 90° - (50° - 30°) = 70°$.

## Prob. 118.

In fig. 70, let $m$Z represent the parallel of declination described by the star.

Then lat. $= PN = 50° 48'$
$\therefore$ $PZ = 39° 12' =$ star's polar distance
$\therefore$ decl. $= 50° 48'$ N.

and alt. below pole $= Nm = NP - Pm$
$= 50° 48' - 39° 12' = 11° 36'$.

## Prob. 119.

Since the sun, in this problem, touches the horizon at the inferior transit, its polar distance must be equal to the latitude of the place (p. 27),

hence lat. $= 90° - 23° 28' = 66° 32'$ N.

*Proof of Formulæ in p.* 33.

(1). The heavenly body on the *prime vertical,* as at Y in fig. p. 25.

Draw the horizon, celestial meridian, and prime vertical; take a point Y on the prime vertical, and draw PY, the polar distance; then

cos. hour-angle = cot. lat. . tan. decl. ($\alpha$)

*Proof.* In triangle PZY, right-angled at Z, mark the hour-angle P, polar distance PY (the co-decl.), and co-lat. PZ; then

cos. P = tan. PZ . cot. PY (Rule XV.)

or cos. hour-angle = cot. lat. . tan. decl.

Again, sin. decl. = sin. lat. . sin. alt. ($\beta$)

*Proof.* In a similar triangle, mark pol. dist. PY, co-lat. PZ, and zen. dist. YZ; then

cos. PY = cos. PZ . cos. YZ

or sin. decl. = sin. lat. . sin. alt.

Again, cos. alt. = sin. hour-angle . cos. decl. ($\gamma$)

*Proof.* In a similar triangle, mark zen. dist. YZ, hour-angle P, and co-decl. PY; then

sin. YZ = sin. P . sin. PY

or cos. alt. = sin. hour-angle . cos. decl.

Again, cos. lat. = cot. hour-angle . cot. alt. ($\delta$)

*Proof.* In a similar triangle, mark zen. dist. YZ, hour-angle P, and co-lat. PZ; then

sin. PZ = cot. P . tan. YZ

or cos. lat. = cot. hour-angle . cot. alt.

(2.) The heavenly body *six hours,* or 90° from the meridian, as at V in fig. p. 25.

Draw the horizon, celestial meridian, and prime vertical, and take a point V six hours from the meridian, and draw PV the pol. dist., and ZV the zen. dist.; then

$$\text{sin. alt.} = \text{sin. decl.} \,.\, \text{sin. lat.} \qquad (\varepsilon)$$

*Proof.* In triangle PVZ, right-angled at P, mark zen. dist. VZ, pol. dist. PV, and co-lat. PZ; then

$$\text{cos. VZ} = \text{cos. PV} \,.\, \text{cos. PZ}$$

$$\text{or sin. alt.} = \text{sin. decl.} \,.\, \text{sin. lat.}$$

Again, cos. azimuth = cot. lat. . tan. alt. $(\zeta)$

*Proof.* In a similar triangle, mark PZV the azimuth, co-lat. PZ, and zen. dist. VZ; then

$$\text{cos. Z} = \text{tan. PZ} \,.\, \text{cot. VZ}$$

$$\text{or cos. azimuth} = \text{cot. lat.} \,.\, \text{tan. alt.}$$

Again, cos. lat. = cot. decl. . cot. azimuth. $(\eta)$

*Proof.* In a similar triangle, mark co-lat., pol. dist., and azimuth; then

$$\text{sin. PZ} = \text{tan. PV} \,.\, \text{cot. Z}$$

$$\text{or cos. lat.} = \text{cot. decl.} \,.\, \text{cot. azimuth.}$$

Again, cos. decl. = sin. azimuth . cos. alt. $(\theta)$

*Proof.* In a similar triangle, mark pol. dist., azimuth, and zen. dist.; then

$$\text{sin. PV} = \text{sin. Z} \,.\, \text{sin. VZ}$$

$$\text{or cos. decl.} = \text{sin. azimuth} \,.\, \text{cos. alt.}$$

(3.) The heavenly body in the *horizon*, as at D, fig. p. 25.

Draw the horizon and celestial meridian as before, and take D a point on the horizon, and draw PD the polar distance, and ZD the zen. dist.; then

$$\text{cos. hour-angle} = -\text{tan. lat.} \,.\, \text{tan. decl.} \qquad (\iota)$$

*Proof.* In quadrantal triangle PZD, mark hour-angle P, co-lat. PZ, and pol. dist. PZ; then (ZD being 90°)

$$\cos. P = -\cot. PZ \,.\, \cot. PD$$

or cos. hour-angle $= -$tan. lat. . tan. decl.

Again, sin. decl. = cos. lat. . sin. amplitude. ($\kappa$)

*Proof.* In a similar triangle, mark PD, PZ, and Z the co-amplitude; then

$$\cos. PD = \sin. PZ \,.\, \cos. Z$$

or sin. decl. = cos. lat. . sin. amplitude.

Again, sin. lat. $= -$cot. amplitude . cot. hour-angle. ($\lambda$)

*Proof.* In a similar triangle, mark PZ, Z, and P; then

$$\cos. PZ = -\tan. Z \,.\, \cot. P$$

or sin. lat. $= -$cot. amplitude . cot. hour-angle.

Lastly, cos. amplitude $=$ cos. decl. . sin. hour-angle. ($\mu$)

*Proof.* In a similar triangle, mark Z, PD, and P; then

$$\sin. Z = \sin. PD \,.\, \sin. P$$

or cos. amplitude = cos. decl. . sin. hour-angle.

In the above formulæ the lat. and decl. are supposed to be of the same name. When they are of different names, the last four formulæ ($\iota$), ($\kappa$), ($\lambda$), ($\mu$), may require some modification: this may be easily done by remembering that PD $= 90° +$ decl. instead of $90° -$ decl., and angle Z $= 90° +$ amplitude instead of $90° -$ ampl., as in former case. The other formulæ require no alteration, since they are applied to problems in which the latitude and declination are always of the same name.

### Prob. 120.

Describe the following parts of fig. p. 25, the horizon, celestial meridian, and prime vertical. Let Y be

the place of the body on the prime vertical, Z the zenith of spectator, and P the pole; then in the spherical triangle PYZ, right-angled at Z, are given, PY (the complement of the decl.) = 70°, and YZ (the complement of alt.) = 60°, to find PZ the complement of latitude, and thence the latitude.

By Rule XV. (*Trig.* Part I.)

$$\cos. PY = \cos. PZ \,.\, \cos. YZ$$

| | |
|---|---|
| log. cos. PY + 10 .. | 19·534052 |
| log. cos. YZ ..... | 9·698970 |
| log. cos. PZ ..... | 9·835082 |

$\therefore$ PZ = 46° 50′ 30″, and lat. = 43° 9′ 30″ N.

PROB. 121.

Describe fig. to prob. 120. In this there are given, colat. PZ = 90° − 50° 48″, or 39° 12′, pol. dist. PY = 90° − 23° 27′ 45″, or 66° 32′ 15″, to find zen. dist. YZ (and thence the alt.), and hour-angle P.

(1).

Mark PY and PZ as known quantities, and YZ as unknown; then, Rule XV.

$$\cos. PY = \cos. PZ \,.\, \cos. YZ$$

| | |
|---|---|
| log. cos. PY + 10, | 19·600045 |
| log. cos. PZ .... | 9·889271 |
| log. cos. YZ .... | 9·710774 |

$\therefore$ YZ = 59° 5′, and alt. = 30° 55′

(2).

Mark PY and PZ as known quantities, and P as unknown; then

$$\cos. P = \cot. PY \,.\, \tan. PZ$$

| | |
|---|---|
| log. cot. PY .. | 9·637524 |
| log. tan. PZ .. | 9·911467 |
| log. cos. P .... | 9·548991 |

$\therefore$ hour-angle = 4h. 37m. 4s.

If the student is acquainted with analytical trigonometry he will simplify the above and similar formulæ that will frequently occur in the following pages, before he opens the table of logarithms. Thus:

since $\cos. PY = \cos. PZ . \cos. YZ$

$\therefore \sin. decl. = \sin. lat. . \sin. alt.$

hence $\sin. alt. = \sin. decl. . \operatorname{cosec}. lat.$

$\cos. P = \cot. PY . \tan. PZ$

or $\cos. hour\text{-}angle = \tan. decl. . \cot. lat.$

From these two expressions the altitude and hour-angle are found directly from the quantities given in the question.

PROB. 122.

Make a right-angled triangle similar to PYZ in fig. p. 25, but so that Y may be to the east of meridian, somewhere on the line ZE; let Y be the place of the star whose altitude YE is 20°; take another point Y′ between Z and Y to represent the place of the other star, whose altitude is 40°, and join PY′; then in triangles PYZ, PY′Z, are given, colat. PZ = 39° 12′, and zenith distances ZY (= 70°) and ZY′ (= 50°), to find the hour-angles at P, and thence YPY′ their difference.

| In triangle ZPY | In triangle ZPY′ |
|---|---|
| $\sin. PZ = \tan. ZY . \cot. ZPY$ | $\sin. PZ = \tan. ZY' . \cot. ZPY'$ |
| whence ZPY = 5h. 8m. 11s. | and ZPY′ = 4h. 8m. 15s. |

∴ difference of hour-angles = 0h. 59m. 56s.

The above formulæ simplified:

$\cot. ZPY = \cos. lat. . \tan. alt.$ $\cot. ZPY' = \cos. lat. . \tan. alt.$

PROB. 123.

Describe horizon, celestial meridian, and prime vertical; take P the pole, as in fig. p. 25, and make the angle VPZ = 6 hours, or 90°; let V be the heavenly body, and join ZV; then in triangle ZPV are given, ZV = 90° − alt., PV = 90° − decl., and P = 90°, to find PZ = 90° − lat.

By Rule XV. $\cos. ZV = \cos. PZ \,.\, \cos. PV$,

whence colat. PZ may be found, and thence the latitude.

Simplified .... sin. alt. = sin. lat. . sin. decl.

$\therefore$ sin. lat. = sin. alt. . cosec. decl.

### Prob. 124.

Construct a similar figure to the one in the preceding problem; then in the right-angled triangle ZPV are given, colat. PZ = 39° 12′, polar dist. PV = 66° 32′ 15″, and P = 90°, to find azimuth Z, and zen. dist. ZV, and thence the altitude.

By Rule XV. $\cos. ZV = \cos. PV \,.\, \cos. PZ$

$\sin. PZ = \tan. PV \,.\, \cot. Z$

Simplified ..... sin. alt. = sin. decl. . sin. lat.

cot. az. = cos. lat. . tan. decl.

### Prob. 125.

Describe the following parts of fig. 97, horizon, celestial meridian, and prime vertical. Let M be the place of the sun at rising, and P the pole; join ZM and PM; then in quadrantal triangle ZPM are given, colat. PZ = 39° 12′, azimuth PZM = 100°, and ZM = 90°, to find hour-angle P, and thence apparent time (def. p. 29).

By Rule XVI. $\cos. PZ = +\tan. PZM \,.\, -\cot. P$

(placing the proper signs over known quantities, by Rule 14, p. 32, Part I., we find P is less than 90°, or 6 hours),

whence P is found = 5h. 28m. 53s.

$\therefore$ app. time = 18h. 31m. 7s., or 6h. 31m. 7s. A.M.

Simplifying formula . . cot. P = sin. lat. . cot. PZM

= sin. 50°48′ . cot. 80°.

PROB. 126.

In fig. p. 25, suppose D the place of the sun on the horizon; then amplitude = WD = 37° 30′. Complete quadrantal triangle PDZ; then in triangle PDZ are given, angle PZD = 52° 30′ (complement of amplitude), pol. dist. PD = 74° 48′, and ZD = 90°, to find colat. PZ, and thence the latitude.

By Rule XVI. $\cos. PD = \underline{\sin. PZ} . \cos. PZD$

whence PZ may be found.

Or simplifying formula. . sin. decl. = cos. lat. . sin. ampl.

$\therefore$ cos. lat. = sin. decl. . cosec. ampl.

PROB. 127.

Construct the following parts of fig. p. 25, horizon, celestial meridian, prime vertical; suppose $D_2$ to be the place where the sun rose, and D the place of setting; join PD and ZD; then in quadrantal triangle PZD there are given, colat. PZ, pol. dist. PD, and ZD = 90°, to find angle PZD (the complement of amplitude), and hour-angle ZPD (the half the length of day), and thence the lengths of the day and night.

By Rule XVI. $\cos. PD = \sin. PZ . \underline{\cos. PZD}$

$\underline{\cos. P} = -\cot. PZ . \cot. PD$

(placing the proper signs over the terms (*Trig.* Part I. p. 32), we shall find cos. PZD is positive, and cos. P negative),

whence PZD = 59° 55′ 30″, and ampl. = W. 30° 4′ 30″ N.

P = 7h. 36m. 41s. = angle $PZD_2$

therefore the sun rises at 4h. 23m. 19s. A.M., sets at 7h. 36m. 41s. P.M., length of day 15h. 13m. 22s., length of night 8h. 46m. 38s., and amplitude at rising and setting 30° 4′ 30″ N.

Formulæ simplified . . sin. ampl. = sin. decl. . sec. lat.
cos. hour-angle = —tan. lat. . tan. decl.

### Prob. 128.

Since the day is more than 12 hours long, the sun must rise (in north lat.) to the north of equator, as at S in fig. 97. Construct fig.; then in quadrantal triangle ZPS are given, colat. PZ = 39° 12′, hour-angle P = 7h., and ZS=90°, to find PZS, the complement of amplitude.

Rule XVI. cos. PZ = —cot. ZPS . tan. PZS
whence PZS = N. 70° 55′ 30″ E.
and ampl. = E. 19° 4′ 30″ N.

Simplified . . cot. ampl. = —sin. lat. . tan. hour-angle.

### Prob. 129.

Describe horizon, celestial meridian, prime vertical, and WQE, the celestial equator, fig. 71. Take some point X on the equator to represent the sun's place, and draw circle of altitude ZO, and circle of decl. PX; then in quadrangle triagle PXZ are given, zenith distance ZX = 67° 4′, hour-angle P=3h., and PX=90°, to find colat. PZ, and thence the latitude.

Rule XVI. cos. ZX = sin. PZ . cos. P
whence PZ = 33° 26′ 30″, and lat. =56° 33′ 30″

Simplified . . cos. lat. = sin. alt. . sec. hour-angle.

### Prob. 130.

Let X, fig. 74, be the place of the heavenly body; then in triangle PXZ are given, zen. dist. ZX, pol. dist. PX, and azimuth PZX, to find colat. PZ, and thence the latitude. The quantities given are not sufficient to

enable us to find PZ by the common rules of spherical trigonometry. In cases of this kind it is usual to drop a perpendicular upon one of the sides, or side produced, thus forming two right-angled triangles, and then to apply, if possible, Rule XV. Problems may often be solved by means of analytical trigonometry, when the common rules are insufficient, or tiresome if applied. We will give a few solutions of this problem, as they will be useful exercises in analytical trigonometry.

(1st solution.) By dropping a perpendicular, as XM, on the meridian.

(1.) In right-angled triangle XZM are given XZ and XZM, to find XM and ZM.

$$\sin. XM = \sin. ZX \,.\, \sin. XZM \quad \therefore\ XM = 38^\circ\,34'\,30''$$
$$\cos. XZM = \cot. XZ \,.\, \tan. ZM \quad \therefore\ ZM = 30\ 12\ 45$$

(2.) In right-angled triangle XPM are given XP and XM, to find PM.

$$\cos. PX = \cos. PM \,.\, \cos. XM \quad \therefore\ PM = 61^\circ\,8'\,45''$$

whence $PZ = PM - ZM = 30^\circ\,56'$, and lat. $= 59^\circ\,4'$ N.

(2d solution.) By means of formula (U), *Trig.* Part II. p. 65.

(1.) Find angle P, by Rule XII., thus:

$$\sin. P : \sin. PZX :: \sin. XZ : \sin. PX$$
$$\therefore\ P = 42^\circ\,19'\,30''$$

(2.) By (U).

$$\tan. \tfrac{1}{2}PZ = \frac{\tan. \tfrac{1}{2}(PX - ZX) \,.\, \sin. \tfrac{1}{2}(PZX + P)}{\sin. \tfrac{1}{2}(PZX - P)}$$

$\therefore\ \tfrac{1}{2}PZ = 15^\circ\,27'\,45''$, and lat. $= 59^\circ\,4'\,30''$ N.

(3d solution.) By means of a subsidiary angle. *Trig.* Part II. p. 64.

$$\cos.\,\mathrm{PZX} = \frac{\cos.\,\mathrm{PX} - \cos.\,\mathrm{PZ}\,.\,\cos.\,\mathrm{ZX}}{\sin.\,\mathrm{PZ}\,.\,\sin.\,\mathrm{ZX}}$$

If $a$ = alt.
$l$ = lat.
$d$ = decl.

$$= \frac{\sin.\,d - \sin.\,l\,.\,\sin.\,a}{\cos.\,l\,.\,\cos.\,a}$$

$$\therefore\ \sin.\,d = \sin.\,a\,.\,\sin.\,l + \cos.\,l\,.\,\cos.\,a\,.\,\cos.\,\mathrm{PZX}$$
$$= \sin.\,a\,(\sin.\,l + \cos.\,l\,.\,\cot.\,a\,.\,\cos.\,\mathrm{PZX})$$

Assume $\cot.\,a\,.\,\cos.\,\mathrm{PZX} = \cot.\,\theta$ . . (1)

$$\therefore\ \sin.\,d = \sin.\,a\,(\sin.\,l + \cos.\,l\,.\,\cot.\,\theta)$$
$$= \frac{\sin.\,a\,(\sin.\,l\,.\,\sin.\,\theta + \cos.\,l\,.\,\cos.\,\theta)}{\sin.\,\theta}$$
$$= \frac{\sin.\,a\,.\,\cos.\,(l \sim \theta)}{\sin.\,\theta}$$

whence $\cos.\,(l \sim \theta) = \sin.\,d\,.\,\sin.\,\theta\,.\,\mathrm{cosec.}\,a$ (2)

$\theta$ is found by (1) = 120° 12′ 45″, and thence, by (2), $l \sim \theta$ = 61° 8′ 30″, and therefore $l$ = 59° 4′ 30″.

*Computation.*

(1). To find $\theta$.

| − | + | − |
|---|---|---|
| $\cot.\,\theta$ | $= \cot.\,a\,.\,\cos.\,\mathrm{PZX}$ | |
| $\cot.\,a$. . . . | 0·037948 | |
| $\cot.\,\mathrm{PZX}$ | 9·727228 | |
| $\cot.\,\theta$. . . . | 9·765176 | |

$\therefore\ \theta = 180° - 59°\,47'\,15''$

(2). To find $l \sim \theta$.

| | |
|---|---|
| $\mathrm{cosec.}\,a$ . . | 0·170317 |
| $\sin.\,d$ . . . . | 9·576689 |
| $\sin.\,\theta$ . . . . | 9·936597 |
| $\cos.\,(l \sim \theta)$ | 9·683603 |

$\therefore\ l \sim \theta = 61°\,8'\,30''$

$\therefore\ l = 59\ \ 4\ 30''$

(4th solution.) By elimination.

Let PX = $p$, ZX = $a_1$, XZM = Z,
PM = $x$, ZM = $y$, XM = $z$;
then colat. = $x - y$.

(1.) To find $y$.

In triangle XZM . . cos. Z = tan. $y$ . cot. $a_1$

$\therefore$ tan. $y$ = cos. Z . tan. $a_1$ = cos. az. . cot. alt. (1)

(2.) To find $x$.

In triangle PXM . . cos. $p$ = cos. $x$ . cos. $z$

In triangle XZM . . cos. $a_1$ = cos. $z$ . cos. $y$

eliminate cos. $z$ by division,

$$\therefore \quad \frac{\text{cos. } p}{\text{cos. } a_1} = \frac{\text{cos. } x}{\text{cos. } y}$$

$$\text{or} \quad \text{cos. } x = \text{cos. } p \text{ . cos. } y \text{ . sec. } a_1$$

$$= \text{sin. decl. . cosec. alt. . cos. } y \qquad (2)$$

hence by (1) $y = 30° 12' 45''$; by (2) $x = 61° 8' 30''$

$\therefore$ $x - y$ the colat. $= 30° 55' 45''$, and lat. $= 59° 4' 15''$.

PROB. 131 (Fig. 74).

In this important problem are given the polar dist. PX$=90°-22° 30'$, the zen. dist. ZX$=90°-37° 20'$, and hour-angle P$=2$h. 15m., to find the latitude$=90°-$PZ.

As in the last problem, the rules in trigonometry will not enable us to find the side PZ from the three known parts of the triangle.

(1st solution.) By dropping a perpendicular XM.

(1.) In right-angled triangle PXM are given PX and P, to find PM.

$$\text{cos. P} = \underline{\text{tan. PM}} \text{ . cot. PX}$$

$$\therefore \quad \text{PM} = 63° 31' 15''.$$

Let $h$ = hour-angle, $d$ = declination, $a$ = alt.

simplified . . . . $\underline{\text{tan. PM}} = \text{cos. } h \text{ . cot. } d$.

(2.) In right-angled triangle PXM are given PX and P, to find MX.

$$\sin.\,MX = \sin.\,P\,.\,\sin.\,PX$$

(or $\sin.\,MX = \sin.\,h\,.\,\cos.\,d$) $\therefore$ MX = 30° 53′.

(3.) In right-angled triangle ZXM are given ZX and XM, to find ZM.

$$\cos.\,ZX = \cos.\,ZM\,.\,\cos.\,MX$$

(or $\cos.\,ZM = \sin.\,a\,.\,\sec.\,MX$) $\therefore$ ZM = 45° 2′ 15″

$\therefore$ PZ = PM−ZM = 18° 29′, and lat. = 71° 31′ N.

(2d solution.) By means of (U), *Trig.*, Part II. p. 65.

(1.) Find angle PZX by Rule XII.

$$\sin.\,PZX : \sin.\,P :: \sin.\,PX : \sin.\,ZX$$

(or $\sin.\,PZX = \cos.\,d\,.\,\sin.\,h\,.\,\sec.\,a$)

$\therefore$ PZX=40° 12′ 30″, or its supplement 139° 47′ 30″, an ambiguous result; but it is evident by a rough construction of figure that the latter value must be taken. See fig. 74.

(2.) By formula (U), find $\frac{1}{2}$PZ (as in last problem) = 9° 14′ 20″, whence lat. = 90°−PZ = 71° 31′ 20″.

(3d solution.) By means of a subsidiary angle.

Since (Part II., p. 64)

$$\cos.\,P = \frac{\cos.\,ZX - \cos.\,PX\,.\,\cos.\,PZ}{\sin.\,PX\,.\,\sin.\,PZ} = \frac{\sin.\,a - \sin.\,d\,.\,\sin.\,l}{\cos.\,d\,.\,\cos.\,l}$$

$\therefore$ $\sin.\,d\,.\,\sin.\,l + \cos.\,d\,.\,\cos.\,l\,.\,\cos.\,P = \sin.\,a$

or $\sin.\,d\,(\sin.\,l + \cos.\,l\,.\,\cot.\,d\,.\,\cos.\,P) = \sin.\,a$

Assume $\cot.\,\theta = \cot.\,d\,.\cos.\,P$ . . . . . (1)

$\therefore$ $\sin.\,d\,(\sin.\,l + \cos.\,l\,.\,\cot.\,\theta) = \sin.\,a$

or $\sin.\,d\,.\,\mathrm{cosec.}\,\theta\,.\,\cos.\,(l \sim \theta) = \sin.\,a$

$\therefore$ $\cos.\,(l \sim \theta) = \sin.\,a\,.\,\mathrm{cosec.}\,d\,.\,\sin.\,\theta$ . . . (2)

Formula (1) determines $\theta$, and thence, by (2), $l \sim \theta$ may be found, and consequently the latitude $l$, as in the last problem.

(4th solution.) By elimination.

Let PX $= p$, ZX $= a_1$, P$=h$,
PM $= x$, ZM $= y$, XM $= z$;

then colat. $= x \mp y$

(1.) To find $x$, or PM.

In triangle XPM .. cos. $h$ = cot. $p$ . tan. $x$

$\therefore$ tan. $x$ = cos. $h$ . tan. $p$

= cos. $h$ . cot. decl. . . (1)

(2.) To find $y$, or ZM.

In triangle XPM .. cos. $p$ = cos. $z$ . cos. $x$

In triangle ZXM .. cos. $a_1$ = cos. $y$ . cos. $z$

eliminate cos. $z$ by division .. $\frac{\text{cos. } p}{\text{cos. } a_1} = \frac{\text{cos. } x}{\text{cos. } y}$

$\therefore$ cos. $y$ = cos. $a_1$ . cos. $x$ . sec. $p$

= sin. alt. . cos. decl. . cos. $x$ . . . (2)

Formulæ (1) and (2) determine $x$ and $y$, and thence the latitude, since colat. $= x \mp y$.

PROB. 132 (Fig. 76).

Ex. 1. Let X be the place of the sun in the ecliptic AXM, A the first point of Aries, and ARQ the celestial equator. Through X draw the circle of declination PXR; then in the spherical triangle AXR, right-angled at R, are given angle A the obliquity of the ecliptic, and AR the sun's right ascension, to find AX the longitude, and XR the declination.

To find longitude AX. Rule XV.

cos. A. = tan. AR . cot. AX $\quad \therefore$ AX $= 64° 33' 15''$

To find declination XR.

sin. AR = tan. XR . cot. A $\therefore$ XR = 21° 4′ 15″ N.

Ex. 2 (fig. 77). In this example the sun's right ascension exceeds 12 hours, it must therefore have passed the autumnal equinox, or first point of Libra, to the south side of the equator.

To find the place of the sun, let $ALA_1$ represent the whole of the celestial equator, A and $A_1$ the same point, namely, the first point of Aries, $ACLC_1A_1$ the ecliptic, cutting the equator in the two points A (or $A_1$) and L. Take AR equal to the sun's right ascension, and through R draw the circle of decl. RX; then X is the place of the sun, and its longitude is ACLX, and decl. RX.

To find longitude ACX, and decl. RX.

Subtract 12 hours from the right ascension ALR, the remainder is LR; and the angle RLX is also known, being the obliquity; then in triangle RLX,

sin. RL = cot. obliq. . tan. decl. $\therefore$ decl. = 22° 58′ S.

cos. obliq. = tan. LR . cot. LX $\therefore$ LX = 78° 30′ 15″

$\therefore$ long. ACLX = 180° + 78° 30′ 15″ = 258° 30′ 15″.

## Prob. 133.

Ex. 1 (fig. 78). Let X be the place of the heavenly body, AR the celestial equator, and AM the ecliptic. Through X draw PXR, a circle of decl., and P′XM, a circle of latitude, and join A and X by an arc of a great circle.

In the figure are given, AR the right ascension, XR the decl., and MAR the obliquity, to find AM the longitude, and XM the latitude.

(1.) In triangle AXR,

cos. AX = cos. RA . cos. XR $\quad \therefore$ AX = 48° 45′ 45″

In same triangle,

sin. AR = tan. XR . cot. XAR $\therefore$ XAR = 29° 7′ 0″

$\therefore$ angle XAM = 29° 7′ 0″ − 23° 27′ 45″ = 5° 39′ 5″

(2.) In triangle XAM,

cos. XAM = cot. AX . tan. AM $\therefore$ long. AM = 48° 37′ 30″

In same triangle,

sin. XM = sin. AX . sin. XAM $\therefore$ lat. XM = 4° 15′ N.

Ex. 2. Since the right ascension exceeds 12 hours, the star must be to the east of the first point of Libra, as at X† in fig. 77. Draw XM a circle of lat., and XR′ a circle of decl., and join LX.

(1.) In right-angled triangle LXR′ are given, LR′ = ♎R.A. − 12h., decl. XR′ = 25° 51′ (N. decl.), to find LX and angle XLR′.

cos. LX = cos. LR′ . cos. XR′ $\quad \therefore$ LX = 66° 19′ 30″

sin. LR′ = cot. XLR′ . tan. R′ $\therefore$ XLR′ = 28° 25′ 45″

(2.) In right-angled triangle XLM, given LX, and angle XLM (= XLR′ + obliquity) = 51° 53′ 30″, to find LM (= longitude − 180°), and XM the latitude.

sin. XM = sin. LX . sin. XLM $\therefore$ XM = 46° 6′ 15″ N.

cos. XLM = cot. LX . tan. LM $\therefore$ LM = 54° 36′ 30″

whence long. ACLM = 180° + 54° 36′ 30″ = 234° 36′ 30″.

## Prob. 134 (Fig. 79).

Let A and B represent the two places in the same latitude, and PU, PV, meridians passing through them,

† In some of the diagrams the star is marked X instead of $X_1$.

and UV the arc of the equator intercepted between the meridians; then UV = difference of longitude between A and B = angle at P = 10° 36′, and AP = PB = colatitude = 45°.

In the spherical triangle APB are given two sides and included angle, to find (Rule X.) the third side AB = 7° 29′ 16″ = 449·25′, the distance between the two places in nautical miles.

Or thus: since AP = PB, a perpendicular drawn from P upon AB will bisect the angle P and arc AB. Suppose in the figure an arc AD to be drawn perpendicular to AB, then in the right-angled triangle PDA are given AP = 45° and angle APD = 5° 18′, to find AD = half the distance AB.

Rule XV..... $\frac{\text{sin. AD}}{\text{sin. AP . sin. APD}}$.

Prob. 135 (Fig. 80).

Let UV be the arc of the equator intercepted between the meridians PB and PV passing through B and A, the two places; join AB. Then in the triangle BPA are given PB = 90° + 34° 37′ = 124° 37′, PA = 90° − 50° 48′ = 39° 12′, and the included angle P = 57° 18′, difference of longitude between the places.

Whence (by Rule X. or XI.) the third side AB may be found = 99° 9′ 48″, or 5949·8 nautical miles.

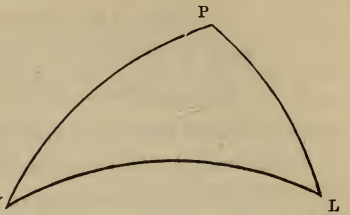
60 nautical miles = 69·05 geographical miles:

hence to express the above distance in geographical miles we have 60 : 69·05 :: 5949·8 : $x$ = 6847·2 geographical or English miles.

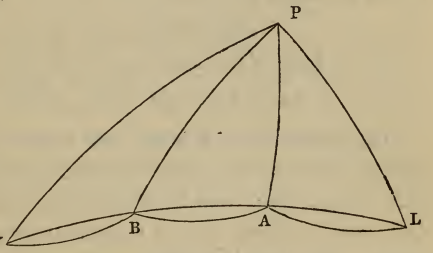
*Great Circle Sailing.*

(1.) Find the shortest distance between Liverpool and New York.

lat. Liverpool . . 53° 25′ N. long. Liverpool . . 2° 59′ W.
„ New York . 40 42 N. „ New York . 73 59 W.




Let PL, PY be the meridians of Liverpool and New York, and LY an arc of a great circle passing through the two places; then in the spherical triangle PYL are given PL = 90°—53° 25′, PY = 90°—40° 42′, and the included angle P = 71°, the difference of longitude between the two places, to find (by Rule X. or XI.) the arc YL = 47° 52′ = 2872 nautical miles.




(2.) Take a point A on the arc YL, distant from Liverpool, L, 872 miles; take another point B, distant from A 1000 miles; then from B to New York, the distance BY = 1000 miles. To find the latitude and longitude of each of the points A and B.

(1.) In the triangle PYL are given the three sides, to find (by Rule VIII. or IX.) the angle L = 75° 8′ 15″.

(2.) To find PA, and thence the latitude of A.

In the triangle PLA are given, the sides AL (=872′ or 14° 32′), PL, and the angle L, just found, to find PA (the colatitude of A) = 35° 21′, and thence the lat. of A = 54° 39′ N.

(3.) To find angle APL, and thence the longitude of A.

In the same triangle APL are given the three sides, to find the angle APL=24° 47′ 15″.

angle APL = 24 47′ 15″ W.
long. of Liverpool = 2 59 0 W.
∴ long. of point A = 27 46 15 W.

(4.) To find PB, and thence the latitude of B.

In the triangle PBL are given the sides BL (=1872′ or 31° 12′), PL, and the angle L, to find PB (the colat. of B) = 40°, and thence the lat. of B = 50° N.

(5.) To find the angle BPL, and thence the longitude of B.

In the same triangle BPL are given the three sides, to find the angle BPL = 51° 10′.

angle BPL = 51° 10′ W.
long. of Liverpool = 2 59 W.
∴ long. of point B = 54 9 W.

The points A and B being thus found on the line of shortest distance, the course and distance from Liverpool to A, thence from A to B, and thence from B to New York, may be found by the common rule of Mercator's sailing. See Rule, p. 151.

The course and distance from Liverpool to New York by Mercator's sailing are S. 75° 10′ 30″ W., 2982 miles, and the sum of the distances described by the ship sailing on the three loops, as above, is 2884, so that the distance saved by altering the course twice is about 98 miles.

The above method of finding the length of the arc of a great circle passing through two places on the surface of the earth, and of determining a certain number of convenient points thereon, to which the ship may be

directed, so as to keep her as near to a great circle as possible, is direct and general. By dropping a perpendicular from P, or by means of special tables, the computation may be shortened, but only with the disadvantage of introducing a distinction of cases, and rendering the problem complicated. As the attempt at great circle sailing can only be of rare occurrence, it does not seem advisable to give formal rules for the purpose, nor to burden our books with more tables than are already in use.

By taking a greater number of points than A and B on the arc YL, so as to bring the ship oftener to the line of shortest distance, the sum of the distances sailed will approximate nearer to the shortest distance. It will be seen, however, that even on the above assumption of only altering the course once in about 1000 miles, the sum of the distances run exceeds by a very few miles the shortest distance, and that the absolute saving in distance between the two points Liverpool and New York is on this supposition nearly 100 miles.

### Prob. 136 (Fig. 81).

Let AQ represent the celestial equator, A the first point of Aries, M the place of the moon, R the place of star; PM, PR circles of declination passing through M and R, and MR the arc of a great circle joining M and R; then MR is the distance required.

In the triangle MPR are given,

PM = 90° + decl. of moon = 95° 19′ 0″

PR = 90° − decl. of star = 76 49 45

and P = diff. of right ascension = 147 46 0

To find MR, the third side. (Rule X. or XI.)

PROB. 137 (Fig. resembling 69).

Construct the figure as follows: describe the horizon NWSE, celestial meridian NS, and prime vertical EW, as in preceding problems. On the meridian take NX $= 6° 10'$, the altitude of the body; then X is the place of the body, and since its decl. $= 13° 10'$, its polar dist. must be $76° 50'$; therefore, from X take a distance XP $= 76° 50'$ towards the zenith; then P is the north pole, and PN the altitude of pole = latitude (p. 27).

Now PN $=$ PX $+$ XN $= 76° 50 + 6° 10'' = 83°$ N.

PROB. 138 (Fig. 84).

Let AQ represent the celestial equator, AR the ecliptic, and S the place of the sun, M the place of moon, P′ the pole of ecliptic, and P′M, P′S circles of latitude passing through M and S. Draw MS an arc of a great circle; then MS is the distance required.

In the quadrantal triangle P′MS are given, P′M $=$ $90°$ − moon's latitude $= 85° 5' 30''$, P′S $= 90°$, and the angle P′ (the difference of longitude) $= 112° 58' 45''$, to find MS.

By Rule XVI., cos. MS $=$ sin. P′M . cos. P′; putting over P′M and P′ their proper signs (see Rule 14, Part I. p. 32), we find cos. MS is negative,

hence MS $= 112° 53' 30''$.

PROB. 139 (Fig. 85).

Let Z be the first position of the ship, and S the point arrived at after describing ZS, an arc of a great circle $= 100'$, or $1° 40'$. Through Z and S draw the meridians ZQ, SQ, meeting in pole P; then the angle SZQ is the initial course $= 45°$, and the angle PSZ, or its opposite angle, is the course of the ship at S. To find this angle, we have in the triangle PZS the two

sides (PZ the colatitude, and ZS the distance run), and the included angle PZS = 135°. Hence the angle at S may be found by Rule XIII.

PROB. 140 (Figs. 86, 87).

In this problem it is not stated whether B is to the east or west of A. As the longitude of C depends on its position with respect to B as well as to A, there will be two solutions. (See figs. 86, 87.)

Let A, B, C, be the three places; then we have given, PA (the colat. of A) = 50°, PB (colat. of B) = 40°, and AB = AC = BC = 20°, to find CP, the colat. of C, and angle CPA, the difference of longitude between A and C, whence the longitude of C may be found.

(1.) In triangle ABP are given the three sides, to find the angle BAP = 51° 48′.

(2.) In triangle BAC are given the three sides, to find the angle BAC = 61° 1′ 30″; hence the angle PAC = 9° 13′ 30″.

(3.) In the triangle PAC are given, the two sides PA and AC, and included angle PAC, to find angle APC (the difference of longitude between A and C) = 6° 13′ 30″, and the side PC (the colat.) = 30° 23′ 15″.

If C is to the east of A (as in fig. 87),
then the long. of C = long. A + 6° 13′ 30″ = 21° 13′ 30″ E.

If C is to the west of A (as in fig. 86),
then the long. of C = long. A − 6° 13′ 30″ = 8° 46′ 0″ E.

PROB. 141 (Fig. 88).

Let T and R represent the places of the stars, C that of the comet, AEQ the ecliptic, and P the pole of ecliptic. Through the heavenly bodies T, C, R, draw the circles of latitude; then CE = lat. of comet, and AE the longitude (A being the first point of Aries).

(1.) In triangle $TP_1R$ are given, the side $P_1T = 90° + \text{lat.} = 95° 28' 45''$, the side $P_1R = 90° - \text{lat.} = 89° 32' 30''$, and the included angle $TP_1R =$ difference of longitudes $= 80° 3' 15''$, to find the third side $TR = 80° 8' 45''$.

(2.) In same triangle $TP_1R$ are given the three sides, to find the angle $P_1TR = 88° 34' 45''$.

(3.) In triangle RTC are given the three sides, to find angle $RTC = 39° 32' 15''$.

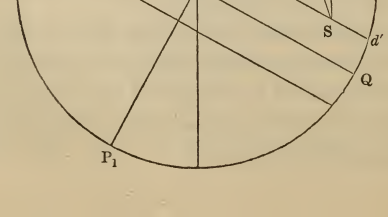
Hence the angle $P_1TC$ is known, being the sum of the last two results $= 128° 7'$.

(4.) In triangle $P_1TC$ are given, $TP_1 = 95° 28' 45''$, $TC = 40° 12'$, and included angle $P_1TC = 128° 7'$, to find $P_1C = 118° 0' 15''$.

Hence CE the lat. $= 28° 0' 15''$ S.

Lastly, in the same triangle $P_1TC$ are given the three sides, to find the angle $TP_1C$ (the difference of long. between comet and Aldebaran) $= 35° 6' 40''$. Hence the longitude $= 67° 12' 15'' + 35° 6' 40'' = 102° 18' 55''$.

PROB. 142.




Let the fig. be projected on the plane of the merid. $PEP_1Q$, Z the zenith of spectator, and H$h$ the plane of horizon. Take HP $= 54° 36'$ then P is the place of the elevated pole (p 27); through P

draw the diameter $PP_1$, and EQ at right angles to it. Then $PP_1$ is the axis of the heavens, and EQ the plane of the celestial equator. Draw $dd'$ parallel to the equator, at the distance from it equal to the sun's declination 8° 30′ N., and let S be the sun's place when twilight commences; draw ZS, PS, arcs of great circles; then S is 18° below the horizon, and ZS = 108°. In the triangle ZPS are given, ZS = 108°, colatitude ZP = 35° 24′, and polar distance PS = 81° 30′, to find the hour-angle ZPS = 9h. 14m. 16s. Hence twilight begins at 2h. 45m. 44s. A.M., and ends at 9h. 14m. 16s. P.M. (the declination being supposed to remain unaltered).

PROB. 143 (Fig. 89).

(1st solution.) Let X and Y be the places of the sun at the times of the observations; PX, PY, the polar distances, each = 66° 34′; and ZX, ZY, the zenith distances = 50° 10′ and 61° 20′; and angle XPY = 1h. 30m. (the interval between the observations). It is required to find PZ the colatitude, and thence the latitude. Draw XY an arc of a great circle.

(1.) In triangle PXY, given PX, PY, each = 66° 34′, and included angle XPY = 1h. 30m., to find XY = 20° 37′ 21″. (Rule X. or XI.)

(2.) In triangle PXY, given the three sides, to find angle PYX = 85° 28′ 30″. (Rule VIII. or IX.)

(3.) In triangle ZYX, given the three sides, to find angle ZYX = 51° 41′.

(4.) Hence PYZ (= PYX − ZYX) = 33° 47′ 30″.

Lastly. In triangle PYZ are given, PY, ZY, and included angle PYZ, to find the third side PZ = 30° 42′ the co-lat. ∴ latitude = 90° − 30° 42′ = 59° 18′ N.

(2d solution [fig. 90]). Let X and Y be the places

of the sun at the times of observation; PX, PY, the polar distances (which in this solution is always supposed to be equal). Bisect XY in M, and join PM, ZM; then PM is at right angles to XY, and therefore the angle PMZ is the complement of ZMX; draw ZE perpendicular to PM.

Let $a_1$, $a_2$ represent the altitudes at X and Y, $h$ the half interval MPY between observations, $d$ the decl., and $l$ the latitude.

Let MX or MY $= x$, PM $= y$, ZE $= u$, PE $= v$, $\therefore$ ME $= y - v$, ZM $= z$, and angle ZMP $= \theta$.

We have to compute the following arcs: (1) $x$; (2) $y$; (3) $u$; (4) $y - v$; thence subtracting arc (4) from arc (2), $v$ is known:* and lastly, knowing $u$ and $v$ in the triangle PZE, PZ the co-lat. can be found.

In triangle PMY .. $\sin. x = \sin. h \,.\, \cos. d$ .... (1)

$\sin. d = \cos. y \,.\, \cos. x \quad \therefore \cos. y = \sin. d \,.\, \sec. x$ .. (2)

In triangle ZME .. $\sin. u = \sin. \theta \,.\, \sin. z$ .... ($\alpha$)

$$\cos. z = \cos. (y - v) \,.\, \cos. u$$

$$\therefore \cos. (y - v) = \cos. z \,.\, \sec. u \ldots\ldots\ldots (\beta)$$

To eliminate $\sin. \theta$ and $\cos. z$ from ($\alpha$) and ($\beta$), we have in the triangle ZMX and ZMY,

$$\cos. \text{ZMX or } \sin. \theta = \frac{\sin. a_1 - \cos. z \,.\, \cos. x}{\sin. z \,.\, \sin. x}$$

* If the great circle drawn through X and Y pass, when produced, between P and Z, the perpendicular ZE will fall without the triangle PZM; in this case ME $= v - y$, and $v$ is found by *adding* arcs (2) and (4) together. Or, since PE must evidently be less than 90°, and the colatitude PZ is always less than 90° (unless the latitude is nothing); consequently when MP+ME *is equal to or greater than* 90°, PE or $v$ cannot be equal to MP+ME; hence $v =$ the *difference* MP−ME; or the difference of arcs (2) and (4) must in such case be taken.

$$\text{cos. ZMY or } -\text{sin. } \theta = \frac{\text{sin. } a_2 - \text{cos. } z \text{ . cos. } x}{\text{sin. } z \text{ . sin. } x}$$

adding $0 = \text{sin. } a_1 + \text{sin. } a_2 - 2 \text{ cos. } z \text{ . cos. } x$

$\therefore$ $2 \text{ cos. } z \text{ . cos. } x = 2 \text{ sin. } \frac{1}{2}(a_1 + a_2) \text{ . cos. } \frac{1}{2}(a_1 - a_2)$

or $\text{cos. } z = \text{sin. } \frac{1}{2}(a_1 + a_2) \text{ . cos. } \frac{1}{2}(a_1 - a_2) \text{ . sec. } x$

Again, subtracting,

$$2 \text{ sin. } \theta = \frac{\text{sin. } a_1 - \text{sin. } a_2}{\text{sin. } z \text{ . sin. } x}$$

$\therefore$ $\text{sin. } \theta = \text{cos. } \frac{1}{2}(a_1 + a_2) \text{ . sin. } \frac{1}{2}(a_1 - a_2) \text{ . cosec. } z \text{ . cosec. } x$

Substituting these values of sin. $\theta$ and cos. $z$ in ($\alpha$) and ($\beta$), we have,

$$\text{sin. } u = \text{cos. } \tfrac{1}{2}(a_1 + a_2) \text{ . sin. } \tfrac{1}{2}(a_1 - a_2) \text{ . cosec. } x \quad \ldots . \quad (3)$$

$$\text{cos.}(y - v) = \text{sin. } \tfrac{1}{2}(a_1 + a_2) \text{ . cos. } \tfrac{1}{2}(a_1 - a_2) \text{ . sec. } x \text{ . sec. } u \quad (4)$$

Lastly, in the right-angled triangle PZE,

$$\text{cos. PZ or sin. } l = \text{cos. } u \text{ . cos. } v.$$

From this solution is deduced the rule in Riddle's Navigation for finding the latitude by double altitude; observing that in finding $y$ we must take the supplement of the arc given by the tables when the lat. and declination are of different names.

From solution 1 is derived the rule for finding the latitude by double altitude given by Dr. Inman. This method is perfectly general, since it may be applied to the same or different heavenly bodies, taken at the same or different times. The rule obtained from the second solution is considered more concise when the same heavenly body is observed, if its declination does not alter in the interval between the observations; as when two altitudes of a star are taken. It will also give a near approximate latitude from two altitudes of the sun; the declination in this case being supposed to remain inva-

riable, and to be taken out of the Nautical Almanac for the middle time between the observations.

PROB. 144 (Fig. 91).

Proceeding in a similar manner as in solution 1 of last problem, we find XY = 22° 42′; PYX = 155° 38′; ZYX = 89° 57′ ∴ PYZ = 65° 41′; PZ = 63° 36′ 53″, whence latitude = 26° 23′ 7″ N.

PROB. 145 *a* (Fig. 92).

Let A and B be the two places in the same latitude, PA, PB, their meridians. Draw PD at right angles to AB, the great circle passing through the two places; then PD bisects the angle APB, since AP = BP; and D is the highest point reached by the ship sailing on a great circle from A to B. Produce also the meridians to the equator UV, and draw the parallel of latitude $AD_1B$.

(1.) In right-angled triangle APD, given PA = 56° 9′, and angle APD = half difference of longitude between A and B = 68° 5′, to find AD = 50° 23′ 45″ and PD = 29° 6′ ∴ lat. of D = 60° 54′, and AB = 2.AD = 100° 47′ 45″, or 6047·5 nautical miles, the distance between A and B on a great circle.

(2.) To find the distance $AD_1B$ on a parallel of latitude, we have (p. 124),

$$AD_1B = UV.\ \cos.\ AU = \text{diff. long.} \times \cos.\ \text{lat.}$$
$$= 8170\ \cos.\ 33° 51′ = 6785 \text{ miles.}$$

whence the difference on the two circles = 737·5 nautical miles.

PROB. 146 (Fig. 93).

Let A and B represent the two places, PA and PB their meridians. Draw the perpendicular PD, then the latitude of D is the highest attained by ship.

(1.) In triangle APB, given PA = 56° 9′, PB = 34° 2′, and angle APB = 140° 27′, to find the angle PAB = 20° 59′ 43″ (Rule XIII.).

(2.) In right-angled triangle PAD, given PA = 56° 9′ and angle PAB = 20° 59′ 45″, to find PD = 17° 18′ 45″ ∴ lat. of D = 72° 41′ 15″.

PROB. 147 (Fig. 83).

Project this figure on the plane of the celestial meridian of the place. Let P be the pole, EQ the celestial equator, Z the zenith, S the place of the sun below the horizon H*h*, PS a circle of declination, and ZS a circle of altitude.

(1.) In triangle ZPS there are given, SP = 90° + sun's decl. = 100° 15′, PZ the colat. = 39° 12′, and hour-angle P = 7h., to find the third side SZ = 107° 24′; whence 107° 24′ − 90° = 17° 24′ the sun's depression.

(2.) In the same triangle the three sides are now known, hence the angle PZS, the azimuth, may be found = N. 84° 53′ W.

PROB. 148 (Fig. 94).

Let X and Y represent the two stars on the same circle of altitude. It is required to find the azimuth of X and Y, or the angle PZX.

(1.) In triangle PXY, given PX = 73° 49′, PY = 61° 35′ 34″, and included angle XPY = 3h. 8m. 27s. (the difference of right ascensions), to find angle PXZ = 65° 46′ 15″.

(2.) In triangle PXZ, given PX, PZ (the colat.), and angle PXZ, to find PZX = 104° 55′; whence its supplement = S. 75° 5′ W. the bearing required.

PROB. 149 (Fig. 95).

Let M and X represent the two bodies on the same

vertical circle ZM; it is required to find PZ, the colat. of the place, and thence the latitude.

(1.) In triangle PXM, given PX = 77° 14′ 15″, PM = 91° 42′ 30″, and included angle XPM = 2h. 36m. 12s. (the difference of right ascensions), to find angle PMZ = 68° 23′ 15″.

(2.) In triangle PMZ, given PM and ZM the moon's zen. dist., and included angle PMZ, to find colat. PZ = 70° 4′ 22″, and therefore lat. = 19° 55′ 38″ N.

### Prob. 150 *a* (Fig. 72).

Suppose the heavenly body to rise at S and to pass the prime vertical at X, then PS = PX the star's polar distance; and EX = star's altitude when due east = 20°. Let $a$ = alt. at X; $m$ = amplitude ES = 11° 15′, $x$ = lat. of spectator, and $y$ = decl. of star.

Then PS = PX = 90° − $y$, PZ = 90° − $x$, and ZX = 90° − $a$.

In quadrantal triangle PZS (Rule XVI.)

$$\sin. y = \cos. x \,.\, \sin. m \quad \ldots\ldots\ldots (1)$$

In right-angled triangle PZX (Rule XV.)

$$\sin. y = \sin. x \,.\, \sin. a \quad \ldots\ldots\ldots (2)$$

∴ equating (1) and (2),

$$\sin. x \,.\, \sin. a = \cos. x \,.\, \sin. m$$

$$\text{or} \quad \frac{\sin. x}{\cos. x} = \frac{\sin. m}{\sin. a}$$

$$\therefore \tan. x = \sin. m \,.\, \operatorname{cosec}. a$$

$$= \sin. 11° 15′ \,.\, \operatorname{cosec}. 20°$$

$$\therefore x = 29° 42′ \text{ N.}$$

### Prob. 151 *a*.

Describe fig. to Prob. 107; take a distance EX = 10°

on the prime vertical, then X will be the place of the heavenly body when on the prime vertical, and $m$ its place on the meridian. Let $XE = a$, and $mS = a_1$, the given altitudes when due east and due south; $l$ = lat., and $d$ = decl. Join PX; in right-angled triangle PZX,

$$\cos. PX = \cos. PZ \,.\, \cos. ZX$$
$$\text{or } \sin. d = \sin. l \,.\, \sin. a \quad \ldots\ldots\ldots\ldots (1)$$

Now $l = ZQ = Zm + mQ = 90° - a_1 + d$

$$\therefore\ d = l + a_1 - 90°$$

$$\therefore\ \sin.(l + a_1 - 90°) = \sin. l \,.\, \sin. a$$
$$\text{or } \sin.(l - \overline{90° - a_1}) = \sin. l \,.\, \sin. a$$
$$\therefore\ \sin. l \,.\, \cos.(90° - a_1) - \cos. l \,.\, \sin.(90° - a_1) = \sin. l \,.\, \sin. a$$
$$\text{or } \sin. a_1 - \cot. l \,.\, \cos. a_1 = \sin. a$$
$$\therefore\ \cot. l \,.\, \cos. a_1 = \sin. a_1 - \sin. a$$
$$= 2 \cos. \tfrac{1}{2}(a_1 + a) \,.\, \sin. \tfrac{1}{2}(a_1 - a)$$

and since $a_1 = 40°$, $a = 10°$,

$$\therefore\ \cot. l = 2 \cos. 25° \,.\, \sin. 15° \,.\, \sec. 40°$$

whence $l = 58°\ 31'$ N.

PROB. 152 *a* (Fig. p. 25).

Let X and V represent the places of the sun when due west and at six o'clock. $a = D_1V$, the alt. at 6 o'clock; $a_1 = WY$, alt. when on prime vertical;

$$\therefore\ ZV = 90° - a, \text{ and } ZY = 90° - a_1$$

Let $x$ = lat., and $y$ = decl., then $PZ = 90° - x$, and $PV = PY = 90° - y$.

(By Rule XV.)

In tri. ZPV, $\cos.(90° - a) = \cos.(90° - y) \,.\, \cos.(90° - x)$

$$\therefore\ \sin. a = \sin. y \,.\, \sin. x \quad \ldots\ldots\ldots (1)$$

In tri. PZY, $\cos.(90° - y) = \cos.(90° - x) \,.\, \cos.(90° - a_1)$

$$\therefore\ \sin. y = \sin. x \,.\, \sin. a_1 \quad \ldots\ldots\ldots (2)$$

Multiplying (1) and (2) together, and cancelling, we have,

$$\sin. a = \sin.^2 x \,.\, \sin. a_1, \text{ or } \sin.^2 x = \sin. a \,.\, \operatorname{cosec}. a_1$$

which formula determines $x$; and dividing (1) by (2), we have,

$$\sin.^2 y = \sin. a \,.\, \sin. a_1$$

whence $y$ may be found.

PROB. 153 *a* (Fig. p. 25).

Let V and D be the places of the body at 6 o'clock and when setting; join PD; then in the triangles PDZ and PVZ are given, $D_1V$ the alt. at 6 o'clock, and DZP the complement of the amplitude, to find the latitude and decl.

Let $a$ = alt. $D_1V$ at 6 o'clock, $m$ = amplitude WD, $x$ = lat. = 90°—PZ, $y$ = decl. = complement of PV or PD.

Then in triangle PDZ . . $\sin. y = \cos. x \,.\, \sin. m$. . (1)

,, ,, PVZ . . $\sin. a = \sin. y \,.\, \sin. x$ . . (2)

Multiplying, and cancelling,

$$\sin. a = \sin. x \,.\, \cos. x \,.\, \sin. m = \tfrac{1}{2} \sin. 2x \,.\, \sin. m$$

$$\therefore \sin. 2x = 2 \sin. a \,.\, \operatorname{cosec}. m$$

whence $2x = 84°$, or $96°$ (the suppl.)

and $x = 42°$, or $48°$

Substituting these values of $x$ in (1), we have

$$y = 22° \ 20' \text{ or } 20°.$$

PROB. 154 *a* (Fig. p. 25).

Let V and D be the places of the body at 6 o'clock and when setting; join PD.

Let $h$ = hour-angle DPZ when setting, $a$ = alt. $D_1V$ at 6 o'clock, $x$ = lat. = 90°—PZ, $y$ = decl. = compl. of PD or PV.

Then (Rules XVI., XV.).

$$\cos.\, h = -\tan.\, y \,.\, \tan.\, x \quad \ldots\ldots\ldots\ldots \quad (1)$$

$$\sin.\, a = \sin.\, y \,.\, \sin.\, x \quad \ldots\ldots\ldots\ldots \quad (2)$$

From (1)

$$\cos.\, h = -\frac{\sin.\, y \,.\, \sin.\, x}{\cos.\, y \,.\, \cos.\, x} = -\frac{\sin.\, a}{\cos.\, y \,.\, \cos.\, x} \text{ (by 2)}$$

$$\therefore \cos.\, y \,.\, \cos.\, x \,.\, \cos.\, h = -\sin.\, a$$

$$\text{and } \cos.\, y \,.\, \cos.\, x = -\sin.\, a \,.\, \sec.\, h$$

But by (2) .. $\sin.\, y \,.\, \sin.\, x = \sin.\, a$

Adding and subtracting, we have

$$\cos.\, x \,.\, \cos.\, y + \sin.\, x \,.\, \sin.\, y = \sin.\, a \,.\, (1 - \sec.\, h)$$

$$\cos.\, x \,.\, \cos.\, y - \sin.\, x \,.\, \sin.\, y = \sin.\, a \,.\, (1 + \sec.\, h)$$

$$\text{or } \cos.\, (x-y) = \sin.\, a \,.\, (1 - \sec.\, h) \,. \quad (3)$$

$$\cos.\, (x+y) = -\sin.\, a \,.\, (1 + \sec.\, h). \quad (4)$$

To reduce (3) and (4) to a logarithmic form.

$$\cos.\, (x-y) = \sin.\, a \,.\, \left(1 - \frac{1}{\cos.\, h}\right) = \frac{\sin.\, a \,.\, (\cos.\, h - 1)}{\cos.\, h}$$

$$= -2 \sin.\, a \,.\, \sin.^2 \frac{h}{2} \,.\, \sec.\, h \quad \ldots\ldots\ldots \quad (5)$$

$$\cos.\, (x+y) = -\sin.\, a \,.\, \left(1 + \frac{1}{\cos.\, h}\right) = -\frac{\sin.\, a \,.\, (\cos.\, h + 1)}{\cos.\, h}$$

$$= -2 \sin.\, a \,.\, \cos.^2 \frac{h}{2} \,.\, \sec.\, h \quad \ldots\ldots\ldots \quad (6)$$

From equations (5) and (6) the values of $x-y$ and $x+y$ may be determined, and thence $x$ and $y$.

### Prob. 155 *a* (Fig. p. 25).

Let $h$ = hour-angle at setting, as at D, $h_1$ = hour-angle at Y on prime vertical, $x$ = lat., $y$ = decl.

Then $\cos.\, h = -\tan.\, x \,.\, \tan.\, y \quad \ldots\ldots\ldots \quad (1)$

$$\cos.\, h_1 = \cot.\, x \,.\, \tan.\, y \quad \ldots\ldots\ldots\ldots \quad (2)$$

Dividing (1) by (2), $\frac{\cos.\ h}{\cos.h_1} = -\frac{\tan.\ x}{\cot.\ x} = -\tan.^2 x$

or $\tan.^2 x = -\cos.\ h\ .\sec.\ h$; which equation determines the value of $x$; and multiplying (1) and (2) together, we have $\tan.^2 y = -\cos.\ h\ .\cos.\ h_1$, whence the declination can be found.

PROB. 156*a* (Fig. p. 25).

Let $h$ = hour-angle at setting, $h_1$ = hour-angle when west, $t$ = interval $\therefore t = h - h_1$, $d$ = decl., $x$ = latitude.

$\cos.\ t = \cos.\ (h - h_1) = \cos.\ h\ .\cos.\ h_1 + \sin.\ h\ .\ \sin.\ h_1$ (1)

By Rule XVI... $\cos.\ h = -\tan.\ x\ .\ \tan.\ d$ ..... (2)

.. .. XV.. $\cos.\ h_1 = \cot.\ x\ .\tan.\ d$..... (3)

Multiplying (2) and (3).. $\cos.\ h\ .\cos.\ h_1 = -\tan.^2 d$

or $2\cos.\ h\ .\cos.\ h_1 = -2\tan.^2 d$

Subtracting equation (1) from this equation,

$\cos.\ h\ .\cos.\ h_1 - \sin.\ h\ .\ \sin.\ h_1 = -\cos.\ t - 2\tan.^2 d$

$\therefore\ -\cos.\ (h + h_1) = \cos.\ t + 2\tan.^2 d$

This equation determines the value of $h + h_1$, the sum of the hour-angles; having this, and the difference, the angles $h$ and $h_1$, and thence the latitude $x$, may be found. The operation is as follows:

To find value of $-\cos.\ (h + h_1) = \cos.\ t + 2\tan.^2 d$.

tab. log. $\cos.\ t = 9{\cdot}864304$ $\therefore$ log. $\cos.\ t = \bar{1}{\cdot}864304$

$\therefore\ \cos.\ t = {\cdot}73165$.

To find value of $2\tan.^2 d$.

log. $(2\tan.^2 d) = \log.\ 2 + 2\log.\ \tan.\ d - 20 = \bar{1}{\cdot}423162$

$\therefore\ 2\tan.^2 d = {\cdot}26495$

$-\cos.\ (h + h_1) = {\cdot}73165 + {\cdot}26495 = {\cdot}9966$

angle corresponding to log. of this in tables $= 0^h\ 18^m\ 54^s$.

Now, since the $\cos.\ (h + h_1)$ is *negative*, the value of $h + h_1$ must be either $12^h - 18^m\ 54^s$ or $12^h + 18^m\ 54^s$.

that is $h+h_1 = 11^h\,41^m\,6^s$ or $12^h\,18^m\,54^s$
and since $h-h_1 = 2\;51\;54$ ,, $2\;51\;54$
$\therefore\ 2h = 14\;33\;0$ or $15\;10\;48$
and $h = 7\;16\;30$ ,, $7\;35\;24$

With the first value of $h$ the lat. $= 42°$
..... other..... $= 48$ nearly.

PROB. 157*a* (Fig. p. 25).

Let Z = angle PZV, the azimuth at six o'clock; $m$ = amplitude WD, $d$ = decl., $l$ = lat.; then
Rule XVI. .... sin. $d$ = cos. $l$ . sin. $m$
.... XV. .... cos. $l$ = cot. $d$ . cot. Z
Multiplying, sin. $d$ . cos. $l$ = cos. $l$ . sin. $m$ . cot. $d$ . cot. Z
$\therefore$ sin.$^2$ $d$ = sin. $m$ . cot. Z . cos. $d$
or $1-\cos.^2 d$ = sin. $m$ . cot. Z . cos. $d$
whence cos.$^2$ $d$ + sin. $m$ . cot. Z . cos. $d = 1$
Completing square, and extracting the root,

$$\cos. d = -\tfrac{1}{2}\sin. m\,.\,\cot. Z + \tfrac{1}{2}\sqrt{\sin.^2 m\,.\,\cot.^2 Z + 4}$$
$$= -0{\cdot}06224 + 1{\cdot}00193 = {\cdot}93969$$

$\therefore$ tab. log. cos. $d = 9{\cdot}972984$ $\therefore$ $d = 20°$
and thence $l$ may be found.

PROB. 158 *a* (Fig. p. 25).

Let $a$ = altitude at 6 o'clock, $m$ = meridian altitude, $d$ and $l$ the decl. and lat.
Then since lat. (QZ) = mer. zen. dist. (Z$m$) + decl.($m$Q)
(see construction of fig. to Prob. 107)
or $l = 90° - m + d$ $\therefore$ $d = l + m - 90°$
$\therefore$ sin. $d$ = sin. $(l+m-90°) = -\sin.(90°-\overline{l+m})$
$= -\cos.(l+m)$
But sin. $a$ = sin. $l$ . sin. $d = -\sin. l$ . cos. $(l+m)$
or 2 sin. $l$ . cos. $(l+m) = -2$ sin. $a$

$$\therefore \text{ sin. } (2l+m) - \text{sin. } m = -2 \text{ sin. } a$$
$$\text{or sin. } (2l+m) = \text{sin. } m - 2 \text{ sin. } a.$$

*Calculation.*

log. sin. $m$ . . . . $\bar{1}\cdot 945935$ $\quad\therefore$ nat. sine . . . . $0\cdot 88295$
„ sin. $a$ . . . . $\bar{1}\cdot 405141$ $\quad\therefore$ nat. sine . . . . $0\cdot 25418$

$\therefore$ sin. $m - 2$ sin. $a = 0\cdot 37459 =$ nat. sin. $(2l+m)$
$\therefore$ tab. log. sin. $(2l+m) = 9\cdot 573556$
$\therefore$ $2l+m = 158°$ (suppl. of $22°$)
whence $l = 48°$ N.

PROB. 159 *a* (Fig. p. 25).

Let $a$ = meridian alt., $h$ = hour-angle at rising or setting, $d$ = decl., $l$ = latitude.

$\therefore$ $d = l - \text{mer. zen. dist.} = l - \overline{90° - a} = l + a - 90°$.

In triangle PZD . . $\cos. h = -\tan. d \,.\, \tan. l$
$$= -\tan. (l+a-90°) \,.\, \tan. l$$
$$= -\tan. -(90° - \overline{l+a}) \,.\, \tan. l$$
$$= \tan. (90° - \overline{l+a}) \,.\, \tan. l$$
$$= \cot. (l+a) \,.\, \tan. l$$
$$= \frac{\cos. (l+a) \,.\, \sin. l}{\sin. (l+a) \,.\, \cos. l}$$

$$\therefore \quad \frac{\cos. h + 1}{\cos. h - 1} = \frac{\cos. (l+a) \,.\, \sin. l + \sin. (l+a) \,.\, \cos. l}{\cos. (l+a) \,.\, \sin. l - \sin. (l+a) \,.\, \cos. l}$$

$$\text{or} \quad \frac{2 \cos.^2 \frac{h}{2}}{-2 \sin.^2 \frac{h}{2}} = \frac{\sin. (2l+a)}{-\sin. a}$$

$$\therefore \cot.^2 \frac{h}{2} \,.\, \sin. a = \sin. (2l+a)$$

whence $2l + a = 29°\ 13'$, or $150°\ 47'$ (its supplement)
Taking this last value,
$$2l = 94°\ 47' \qquad \therefore \ l = 47°\ 23'\ 30''$$

To avoid the ambiguity arising from obtaining the value of $2l+a$ in terms of the sine, let $z = 90°-a$ the zenith dist., then $d = l-z$

$$\therefore \cos.\, h = -\tan.\, d\,.\tan.\, l = -\tan.\,(l-z)\,.\tan.\, l$$

$$= \frac{-2\sin.\,(l-z)\,.\sin.\, l}{2\cos.\,(l-z)\,.\cos.\, l} = \frac{\cos.\,(2l-z)-\cos.\, z}{\cos.\,(2l-z)+\cos.\, z}$$

$$\therefore \quad \frac{\cos.\, h+1}{\cos.\, h-1} = \frac{\cos.\,(2l-z)}{-\cos.\, z}$$

$$\text{or } \cos.\,(2l-z) = \cos.\, z\,.\cot.^2\frac{h}{2}$$

from which equation the value of $l$ may be obtained free from ambiguity.

PROB. 160 *a* (Fig. p. 25).

Let $h$ = hour-angle at setting

$h_{,}$ = .. .. when west.

By Rule XVI. .... $\cos.\, h = -\tan.\, l\,.\tan.\, d$

.. .. XV. .... $\cos.\, h_{,} = \cot.\, l\,.\tan.\, d$

adding, $\cos.\, h+\cos.\, h_{,} = (\cot.\, l-\tan.\, l)\,.\tan.\, d$

subtracting, $\cos.\, h-\cos.\, h_{,} = -(\tan.\, l+\cot.\, l)\,.\tan.\, d$

dividing, $$\frac{\cos.\, h+\cos.\, h_{,}}{\cos.\, h-\cos.\, h_{,}} = -\frac{\cot.\, l-\tan.\, l}{\tan.\, l+\cot.\, l}$$

or $$\frac{\cos.\, h+\cos.\, h_{,}}{-(\cos.\, h-\cos.\, h_{,})} = \frac{\cot.\, l-\tan.\, l}{\tan.\, l+\cot.\, l}$$

$$\therefore \frac{2\cos.\,\frac{1}{2}(h+h_{,})\,.\cos.\,\frac{1}{2}(h-h_{,})}{2\sin.\,\frac{1}{2}(h+h_{,})\,.\sin.\,\frac{1}{2}(h-h_{,})} = \frac{1-\tan.^2 l}{\tan.^2 l+1} = \cos.\, 2$$

$$\text{or } \cot.\,\tfrac{1}{2}(h+h_{,})\,.\cot.\,\tfrac{1}{2}(h-h_{,}) = \cos.\, 2l$$

$$\therefore \quad \cot.\,\tfrac{1}{2}(h+h_{,}) = \cos.\, 2l\,.\tan.\,\tfrac{1}{2}(h-h_{,})$$

which equation determines $\frac{1}{2}(h+h_{,})$, and with $\frac{1}{2}(h-h_{,})$ already known, the angles $h$ and $h_{,}$ may be found, and thence by Rule XV. we can find $d$.

PROB. 161 *a* (Fig. 96).

Suppose the heavenly body to rise at S, and to describe the parallel of declination SX. Let a circle of altitude ZO touch the parallel SX at X; then X is the point where the azimuth is the greatest; the azimuth increasing from the point S, where the heavenly body rose, to X, and beginning to decrease after passing X. The angle PXZ is evidently a right angle; whence the required azimuth, hour, and altitude may all be found from the right-angled triangle PZX.

(1.) To find greatest azimuth PZX.

$\cos. d = \cos. l . \sin. \text{az.} \qquad \therefore \quad \sin. \text{az.} = \cos. d . \sec. l$

(2.) To find the hour-angle ZPX, or $h$.

$\cos. h = \cot. d . \tan. l.$

(3.) To find altitude XO or $a$.

$\sin. l = \sin. d . \sin. a \qquad \therefore \quad \sin. a = \sin. l . \operatorname{cosec}. d$

PROB. 162 *a* (Fig. 96).

The preceding problem furnishes an explanation of the fact, that when the sun's declination is greater than the latitude of a place, and of the same name with it, the shadow of an upright object on a horizontal plane goes backward each day during a certain period, which may be computed, as well as the arc through which it goes back.

At the point X the sun appears stationary in azimuth.

PZS is its bearing when rising

PZO .. .. .. at point X.

The angle SZO or arc SO will therefore denote the number of degrees the bearing has increased from sun rising, or that the shadow of the object has gone back;

and the period during which the retrograde motion has taken place will be measured by the angle SPX.

(1.) To find bearing PZS at rising.

In triangle PZS, $\sin. d = \cos. l . \cos. \text{PZS}$
whence $\text{PZS} = 64° 55' 30''$
By last prob. $\text{PZX} = 77\ 28\ 0$

$\therefore$ shadow goes back $= 12\ 32\ 30$

(2.) To find hour-angle ZPS at rising.

$$\begin{array}{lrrr} & - & + & + \\ \text{In triangle PZS,} & \cos. \text{ZPS} = & -\tan. l . \tan. d & \\ & \therefore \ \text{ZPS} = & 6\text{h.}\ 36\text{m.}\ 22\text{s.} & \\ \text{by last. prob.} & \text{ZPX} = & 2\ \ 12\ \ 7 & \end{array}$$

$\therefore$ time of shadow's going back $= 4\ \ 24\ \ 15$

PROB. 163 *a* (Fig. 97).

(1st solution.) Let S and M be the respective places of the sun and moon at the time of observation; $d$ and $-d$ the decl. of sun and moon respectively; that of the moon being taken negatively, since the bodies are on different sides of the equator; $h$ and $h_,$ their hour-angles.

Then $h - h_, = 1\text{h.}\ 53\text{m.}\ 42\text{s.}$

In triangle ZPS, $\cos. h = -\tan. l . \tan. d$ ..... (1)
In triangle ZPM, $\cos. h_, = -\tan. l . \tan. (-d_,)$
$= \tan. l . \tan. d_,$ ..... (2)

dividing (2) by (1), $\dfrac{\cos. h_,}{\cos. h} = \dfrac{\tan. d_,}{-\tan. d}$

or $\dfrac{\cos. h_, + \cos. h}{\cos. h_, - \cos. h} = \dfrac{\tan. d_, - \tan. d}{\tan. d_, + \tan. d}$

$\therefore \dfrac{2 \cos. \frac{1}{2}(h_, + h) . \cos. \frac{1}{2}(h_, - h)}{-2 \sin. \frac{1}{2}(h_, + h) . \sin. \frac{1}{2}(h_, - h)} = \dfrac{\sin. d_, . \cos. d - \cos. d_, . \sin. d}{\sin. d_, . \cos. d - \cos. d_, . \sin. d}$

or $-\cot.\frac{1}{2}(h_{,}+h).\cot.\frac{1}{2}(h_{,}-h)=\dfrac{\sin.(d_{,}-d)}{\sin.(d_{,}+d)}$

$\therefore\ \cot.\frac{1}{2}(h_{,}+h)=-\sin.(d_{,}-d).\operatorname{cosec}.(d_{,}+d).\tan.\frac{1}{2}(h_{,}-h)$

$=\sin.(d_{,}-d).\operatorname{cosec}.(d_{,}+d).\tan.\frac{1}{2}(h-h_{,})$

whence $\frac{1}{2}(h_{,}+h)=5\text{h. }53\text{m. }29\text{s.}$

and since $\frac{1}{2}(h-h_{,})=0\ \ 56\ \ 51$

$\therefore\ \ h=6\ \ 50\ \ 20$

hence app. time $=12\text{h.}-6\text{h. }50\text{m. }20\text{s.}=5\text{h. }9\text{m. }40\text{s. A.M.}$

To find lat. $l$. $\quad -\qquad +\qquad +$

$\cos.h=-\tan.l.\tan.d$

or $\tan.l=\cos.h.\cot.d$

$\therefore\ \ l=50^\circ\ 18'\ 20''\ \text{N.}$

(2d solution.) More easily, thus:

(1.) In triangle PMS, given PS, PM, and included angle SPM, to find angle PSM $= 128^\circ\ 33'\ 30''$.

(2.) In right-angled triangle NPS, given PS and angle PSN (supplement of PSM), to find lat. NP $=$ $50^\circ\ 18'\ 20''$ N.

### Prob. 164 *a*.

Let $h$ and $h_{,}$ denote the hour-angles of the sun at rising or setting at the two places: then since it rises and sets at the same instant of absolute time at the two places, the difference of the hour-angles $h$ and $h_{,}$ or $h-h_{,}$ must be equal to the difference in their reckoning of time, or to their difference of long. (in time) $= 28^\circ\ 46'$.

Let $l$ and $-l_{,}$ denote the latitudes of Dublin and Pernambuco: the latitudes being affected with different signs, since the places are on opposite sides of the equator.

Then $\cos.h=-\tan.l.\tan.d$ . . . . . (1)

$\cos.h_{,}=-\tan.(-l_{,}).\tan.d=\tan.l_{,}.\tan.d$ . . (2)

$$\therefore \quad \frac{\cos.h_{,}}{\cos.h}=\frac{\tan.l}{-\tan.l_{,}}, \quad \text{or} \quad \frac{\cos.h_{,}+\cos.h}{\cos.h_{,}-\cos.h}=\frac{\tan.l_{,}-\tan.l}{\tan.l_{,}+\tan.l}$$

$$\therefore \quad \frac{\cos.\frac{1}{2}(h_{,}+h).\cos.\frac{1}{2}(h_{,}-h)}{-\sin.\frac{1}{2}(h_{,}+h).\sin.\frac{1}{2}(h_{,}-h)}=\frac{\sin.(l_{,}-l)}{\sin.(l_{,}+l)}=\frac{-\sin.(l-l_{,})}{\sin.(l+l_{,})}$$

$$\text{or} \quad \cot.\tfrac{1}{2}(h_{,}+h)=-\sin.(l-l_{,}).\operatorname{cosec}.(l+l_{,}).\tan.\tfrac{1}{2}(h-h_{,})$$

This equation determines half the sum of the hour-angles $= 101^\circ\ 40'\ 30''$, and thence, with $h-h_{,}$ already known, the values of $h$ or $h_{,}$, and by (1) the declination $d = 18^\circ\ 6'$. Then the days on which the declination of the sun is the value of $d$ thus found will be those required.

PROB. 165 *a*.

Let O be the center of a square ABCD, each of whose sides is 300 feet; draw obelisk OP perpendicular to the plane of ABCD, and make it equal to 60 feet; join AO and AP; then the angle PAO measures the altitude of the sun when the shadow AO of obelisk reaches to A the corner of the square.

To find the angle PAO.

Draw OE perpendicular to AD; then

$$\text{AE} = \text{EO} = \tfrac{1}{2}\text{AD} = 150 \text{ feet}.$$

$$\therefore \ \text{AO} = \text{AE}\sqrt{2} = 150\sqrt{2}.$$

$$\text{and tan. PAO} = \frac{\text{OP}}{\text{AO}} = \frac{60}{150\sqrt{2}} = \frac{6}{30}\sqrt{2} = \frac{\sqrt{2}}{5} = \cdot 2\sqrt{2}$$

whence angle PAO $= 15^\circ\ 47'\ 30''$, the sun's altitude.

PROB. 166 *a* (Fig. 98).

Let ZQH represent the celestial meridian, H the horizon, Z the zenith, Q the equator, S′ and S the places of the sun on the longest and shortest days.

Then ZQ = lat. ($x$), and QH = $90°-x$
SQ = S′Q = decl. ($d$) = 23° 28′

Let NG (the shadow of object PN when sun is at S′) = $y$; then NC (its shadow when at S) = $7y$.

Now angle NPC or arc ZS = $x+d$
and angle NPG or arc ZS′ = $x-d$

In triangle NPC .. $7y$ = PN . tan. NPC = PN . tan. $(x+d)$
.. .. NPG .. $y$ = PN . tan. NPG = PN . tan. $(x-d)$

dividing .. $$7=\frac{\tan.(x+d)}{\tan.(x-d)}=\frac{\sin.2x+\sin.2d}{\sin.2x-\sin.2d}$$

$$\therefore\ \frac{8}{6}\text{ or }\frac{4}{3}=\frac{\sin.2x}{\sin.2d}\quad\text{or}\quad\sin.2x=\frac{4}{3}\sin.2d$$

whence $x$ = 38° 27′ 45″ = lat.

PROB. 167 *a* (Fig. 98).

Let NG = $x$, NC = $y$, $l$ = lat., $d$ = decl.
then $x$ = PN . tan. $(l-d)$, $y$ = PN . tan. $(l+d)$

$$\therefore\ \frac{y}{x}=\frac{\tan.(l+d)}{\tan.(l-d)}=\frac{\sin.2l+\sin.2d}{\sin.2l-\sin.2d}$$
$$=\frac{\sin.90°+\sin.30°}{\sin.90°-\sin.30°}=\frac{1+\frac{1}{2}}{1-\frac{1}{2}}=\frac{3}{1}$$
or $y : x :: 3 : 1$.

PROB. 168 *a*.

In fig. 98, if S′ be the place of the sun, and NG the shadow of object PN, then

$$\frac{\text{PN}}{\text{NG}}=\frac{3}{5}=\tan.\text{PGN}=\tan.\text{sun's alt.}=\cdot6$$

$\therefore$ log. tan. alt. = 9·778151 $\therefore$ alt. = 30° 58′.

Then in triangle PZX (fig. 73), the three sides are given, viz. PZ = 56° 30′, PX = 79° 45′, and ZX = 52° 2′,

to compute the hour-angle ZPX, or $h$; hence $h=$ 3h. 58m. 4s.

PROB. 169 *a* (Fig. 99).

Let AB represent the shadow of perpendicular stick AD, and AC its shadow when the stick is placed in the position AT perpendicular to ray SC, in which position it will manifestly throw the greatest shadow. Now the distance of S is so great compared with BC that the angle BSC has no assignable magnitude, and the rays SB, SC may therefore be considered parallel, or the angle ABD = angle ACT = sun's altitude required.

In triangle ABD . . AD = AB . tan. alt. = 4 tan. alt.

In ATC . . AT = AD = AC . sin. alt. = 5 sin. alt.

$\therefore$ 5 sin. alt. = 4 tan. alt. = 4 sin. alt. . sec. alt.

$\therefore$ sec. alt. = $\frac{5}{4}$ = 1·25

log. sec. alt. = 10·096910

$\therefore$ alt. = 36° 52′ 15″.

PROB. 170 *a* (Fig. 100).

Let A be the observer, B the shadow of cloud, C and D the place of cloud and sun. Since the distance AB will subtend no sensible angle at the sun, the rays AD and BD may be considered parallel, and angle CBE = DAB= sun's alt. = 22°, and the angle of elevation of cloud = 20°. Therefore ACB = 2°.

In tri. ABC . . $\frac{BC}{AB} = \frac{\text{sin. } 20°}{\text{sin. } 2°}$ $\therefore$ BC = $\frac{AB \text{.sin. } 20°}{\text{sin. } 2°}$

In right-angled triangle CBE. . CE = CB . sin. 22°

$\therefore$ CE = AB . sin. 20°. sin. 22°. cosec. 2° = 1468.

PROB. 171 *a* (Fig. 101).

Let $h$ denote the hour-angle of the sun at rising, or

half the length of the day; and since the meridian altitude is less than the latitude, the declination must be of a different name to latitude, or south.

In quadrantal triangle PZS,

cos. $h = -\text{tan.}\, l \,.\, \text{cot.}\, (90° + d) = \text{tan.}\, l \,.\, \text{tan.}\, d = \text{tan.}\, d$, since tan. $l = \text{tan.}\, 45° = 1$.

$$\therefore \text{cos.}\, h = \text{tan.}\, (z - l) \quad \text{if } z = \text{zen. dist.}$$

$$= \frac{\text{tan.}\, z - \text{tan.}\, l}{1 + \text{tan.}\, z \,.\, \text{tan.}\, l} = \frac{\text{tan.}\, z - 1}{\text{tan.}\, z + 1}$$

$$\therefore \frac{\text{cos.}\, h + 1}{\text{cos.}\, h - 1} = -\text{tan.}\, z \quad \text{or} \quad \frac{1 + \text{cos.}\, h}{1 - \text{cos.}\, h} = \text{tan.}\, z$$

$$\therefore \text{cot.}^2 \frac{h}{2} = \text{tan.}\, z = \text{cot. alt.} = \text{cot.}\, 30° = \sqrt{3}$$

$$\text{hence cot.}\, \frac{h}{2} = \sqrt[4]{3} \text{ and tan.}\, \frac{h}{2} = \frac{1}{\sqrt[4]{3}}$$

PROB. 172 *a*.

Let $z$ = zenith dist. $\therefore z = 2l$

and (fig. 66) $z = l + d$ $\therefore 2l = l + d$ or $l = d$.

In triangle PZS (fig. 101),

$$\text{cos.}\, h = -\text{tan.}\, l \,.\, \text{cot.}\, (90° + d)$$

$$= \text{tan.}\, l \,.\, \text{tan.}\, d = \text{tan.}^2 l$$

$\therefore$ tan.$^2 l$ = cos. 7h., whence $l = 26°\ 58'$.

PROB. 173 *a* (Fig. p. 25).

Suppose $D_2$ the place where the sun rose; draw $PD_2$ and $ZD_2$; let $h$ = hour-angle, and $z$ = meridian zenith distance, $l$ = lat., and $d$ = decl. Then

cos. $h = -$ tan. $l$ . tan. $d = -$tan. $d$ since $l = 45°$

but $d = l - z$ $\therefore$ cos. $h = -$tan. $(l - z)$

$$= -\frac{\text{tan.}\, l - \text{tan.}\, z}{1 + \text{tan.}\, l \,.\, \text{tan.}\, z} = \frac{\text{tan.}\, z - 1}{\text{tan.}\, z + 1}$$

$$\text{or} \quad \frac{\text{cos. } h+1}{\text{cos. } h-1} = -\text{tan. } z = -\text{ cot. alt.}$$

$$\therefore \text{ cot. alt.} = \text{cot.}^2 \frac{h}{2} = \text{cot.}^2 60° = \frac{1}{3}$$

$$\therefore \text{ tan. alt.} = 3.$$

PROB. 174 *a* (Fig. 102).

Let $S_,$ be the apparent place of the sun when its upper limb touches the horizon, S its apparent place when lower limb touches horizon; then angle $S_,PS$ measures the time elapsed between the upper and lower limbs being in horizon.

$$S_,PS = ZPS_, - ZPS.$$

(1.) In triangle $ZPS_,$, given $ZP = 40°$, $ZS_, = 90° 16'$, and $PS_, = 84° 22'$ $\therefore$ angle $ZPS_,$ = 6h. 28m. 41s.

(2.) In triangle ZPS, given $ZP = 40°$, $ZS = 90° - 16' = 89° 44'$, and $PS = 84° 22'$ $\therefore$ ZPS = 6h. 25m. 22s.

$\therefore$ angle $S_,PS$ = 3m. 19s.

PROB. 175 *a* (Fig. 103).

Let AB represent the arc of the parallel of decl. described by sun's center, P the pole of the heavens; then considering the triangle APB as a spherical triangle (which may be done in this case, since arc AB being small does not differ much from the corresponding arc of a great circle), we have the three sides of APB given to find the angle P, the time of semidiameter passing meridian. Now (*Trig.* Part II. p. 64)

$$\text{sin.}^2 \frac{P}{2} = \text{cosec. AP . cosec. PB . sin.} \tfrac{1}{2}(\text{AB} + \text{AP} - \text{PB}) \text{ . sin.} \tfrac{1}{2}(\text{AB} - \overline{\text{AP} - \text{PB}}).$$

$$= \text{cosec.}^2 \text{ AP . sin.}^2 \text{ AB} \quad \text{since AP} = \text{BP}$$

$$\therefore \text{ sin.} \frac{P}{2} = \text{sec. } d \text{ . sin. } 8' \, 8{\cdot}6''$$

and P = 1m. 11s. nearly.

Or thus: Let AB represent an arc of a small circle equal in length to the sun's semidiameter, A the sun's center. Through A and B draw circles of declination PU, PV, cutting the equator in U and V; then the time of the point A describing the arc AB will be measured by the arc UV, or the angle UPV.

Now (*Trig.* Part II. p. 85)

$$UV = AB \cdot \sec. AU = 16' 17{\cdot}3''. \sec. d$$

$$\text{or arc UV (in time)} = \frac{977{\cdot}3}{15} \sec. d = 1\text{m. } 10{\cdot}22\text{s.}$$

(this is independent of the sun's apparent motion from west to east).

PROB. 176 *a* (Fig. 104).

Let A be the first station of ship, B the second, and C the point of land.

(1.) In triangle ABC, given AB = 6 miles, angle CAB (the difference of bearing between C and B) = 2 points, and angle ACB = $4\frac{1}{2}$ points, to find AC.

(2.) Draw AD due east, and CD at right angles to it, then CD = difference of latitude between A and C, and (considering ADC as a right-angled plane triangle) CD = AC . sin. CAD = AC . sin. 4 points = 5′ 15″

$\therefore$ lat. of C = 59° 6′ + 5′ 15″ = 59° 11′ 15″

also (the triangle ADC being isosceles) AD = 5′ 15″.

To find the difference of longitude between A and C, we have (p. 160) AD = diff. long. . cos. lat. A, whence diff. long. = 10′ 13″ E.

$\therefore$ long. of C = 6° 15″ + 10′ 13″ = 6° 25′ 13″ E.

PROB. 177 *a* (Fig. 97).

Let S and M be the places of sun rising on the longest and shortest days,

$h$ = hour-angle of sun at S,

$h_1$ = ..... M;

then 2 $(h-h_1)$ = difference in length of day.

In triangle ZPS .. cos. $h = -$tan. $d$ . tan. $l$ ..... (1)

..... ZPM.. cos. $h_1 = +$tan. $d$ . tan. $l$ ..... (2)

$\therefore$ cos. $h =$ cos. $h_1 =$ cos. $(12\text{h.} - h_1)$

$\therefore \; h = 12 - h_1$

or $h - h_1 = 12 - 2h_1 = \frac{1}{2}$ difference required.

Now in (2), cos. $h_1$ = tan. $d$ . tan. $l$ = tan. $d$, since $l = 45°$.

*Calculation.*

log. tan. $d$ .... 9·637611 = log. cos. $h_1$

$\therefore \; h_1 =$ 4h. 17m. 5s., and $2h_1 =$ 8h. 34m. 10s.

$\therefore \; h - h_1 \; (= 12 - 2h_1) =$ 3h. 25m. 50s.

and difference in length of days = 6h. 51m. 40s.

PROB. 178 *a* (Fig. 97).

Let $h$ = hour-angle of the sun at S,

$h_1$ ..... M $\quad \therefore \; h = h_1 + 3$.

In triangle ZPS.... cos. $(h_1 + 3) = -$tan. $d$ . tan. $l$ . (1)

..... ZPM ..... cos. $h_1 =$ tan. $d$ . tan. $l$ .... (2)

$\therefore$ cos. $(h_1 + 3) = -$cos. $h_1 =$ cos. $(12 - h_1)$

$\therefore \; h + 3 = 12 - h_1$ or $h_1 =$ 4h, 30m.

whence, by (2), $l = 46° \; 46'$.

PROB. 179 *a*.

Let $h$ and $h_1$ be the hour-angle; then $h - h_1 =$ 1h.

$d$ and $d_1$ the decl. at these times, viz. 20° and 10°

then cos. $h = -$tan. $l$ . tan. $d$ ..... (1)

cos. $h_1 = -$tan. $l$ . tan. $d_1$ ..... (2)

$$\therefore \; \frac{\text{cos. } h}{\text{cos. } h_1} = \frac{\text{tan. } d}{\text{tan. } d_1} \text{ or } \frac{\text{cos. } h + \text{cos. } h_1}{\text{cos. } h - \text{cos. } h_1} = \frac{\text{tan. } d + \text{tan. } d_1}{\text{tan. } d - \text{tan. } d_1}$$

$$\text{or } \frac{2\cos.\frac{1}{2}(h+h_{,})\,.\,\cos.\frac{1}{2}(h-h_{,})}{-2\sin.\frac{1}{2}(h+h_{,})\,.\,\sin.\frac{1}{2}(h-h_{,})} = \frac{\sin.(d+d_{,})}{\sin.(d-d_{,})}$$

$$\therefore\ -\cot.\tfrac{1}{2}(h+h_{,})\,.\cot.\tfrac{1}{2}(h-h_{,}) = \frac{\sin.30^\circ}{\sin.10^\circ}$$

$$\begin{array}{cccc} - & + & + & + \end{array}$$

$$\therefore\ -\cot.\tfrac{1}{2}(h+h_{,}) = \tan.7^\circ 30'.\sin.30^\circ.\operatorname{cosec}.10^\circ$$

whence $\frac{1}{2}(h+h_{,}) = 7\text{h. } 23\text{m. } 3\text{s.}$

and $\frac{1}{2}(h-h_{,}) = 0 \quad 30 \quad 0$

$$\therefore\ h = 7 \quad 53 \quad 3$$

with this value of $h$, the lat. (by 1) $= 52^\circ\ 27'$ N.

PROB. 180 *a* (Fig. 97).

Let $h$ be the hour-angle of sun at S,

$h_{,}$ ..... M $\quad\therefore\ h = 3h_{,}$

then $\cos.h = -\tan.l.\tan.d$ ..... (1)

$\cos.h_{,} = +\tan.l.\tan.d$ ..... (2)

$\therefore\ \cos.h = -\cos.h_{,} = \cos.(12\text{h.} - h_{,})$

$\therefore\ h = 12 - h_{,}$ or $3h_{,} = 12 - h_{,}$ $\quad\therefore\ 4h_{,} = 12$

and $h_{,} = 3$ hours; whence, by (2) $\quad l = 58^\circ\ 27'$ N.

PROB. 181 *a*.

Let $h$ and $h_{,}$ be the hour-angles,

then $h_{,} = \frac{3}{5}h$

and $\cos.h = -\tan.d.\tan.l$

$\cos.h_{,} = \tan.d.\tan.l$

$\therefore\ \cos.h = -\cos.h_{,} = \cos.(12 - h_{,})$

$$\therefore\ h = 12 - h_{,} = 12 - \frac{3}{5}h$$

$$\therefore\ \frac{8h}{5} = 12 \quad \text{and } h = \frac{60}{8} = 7\text{h. } 30\text{m.}$$

with this value of $h$ the lat. $= 41^\circ\ 24'$ N.

PROB. 182 (Fig. 105).

Let A be the place sailed from, B the place arrived at; the arc $AA_, =$ arc $A_,B = a$, and $BC = b$; EQ the arc of equator between meridians of A and $A_,$.

Let $x =$ lat. of A; then $x-a =$ lat. of B.

*Trig.* Part II. p. 85,

$$\frac{CB}{EQ}=\frac{b}{EQ}=\text{cos. BQ}=\text{cos. }(x-a)$$

$$\frac{AA_,}{EQ}=\frac{a}{EQ}=\text{cos. AE}=\text{cos. }x$$

$$\therefore\ \frac{b}{\text{cos. }(x-a)}=\frac{a}{\text{cos. }x}$$

$$\text{or } \frac{b}{a}=\frac{\text{cos. }(x-a)}{\text{cos. }x}=\frac{\text{cos. }x\,.\,\text{cos. }a+\text{sin. }x\,.\,\text{sin. }a}{\text{cos. }x}$$

$$=\text{cos. }a+\text{tan. }x\,.\,\text{sin. }a$$

$$\therefore\ \frac{\frac{b}{a}-\text{cos. }a}{\text{sin. }a}=\text{tan. }x.$$

*Calculation.*

$b = 150,\ a = 100.$

$$\text{tan. }x=\frac{1{\cdot}5-\text{cos. }1^\circ 40'}{\text{sin. }1^\circ 40'}$$

log. cos. $1^\circ 40'$.. $\bar{1}{\cdot}999816$ $\quad\therefore$ cos. $1^\circ 40'$.. $\cdot 9994$

$$\text{tan. }x=\frac{1{\cdot}5-{\cdot}9994}{\text{sin. }1^\circ 40'}={\cdot}5006\,.\,\text{cosec. }1^\circ 40'$$

$\therefore\ x$ or lat. A $= 86^\circ 40'\ 30''$ N.

difference of lat. $=\ 1\ 40\ \ 0$ S.

lat. of B required $= 85\ \ 0\ 30$ N.

PROB. 183 (Fig. 106).

Let XZ be the apparent zenith distance of sun

SZ true .. .. ..

O the object whose bearing is required.

(1.) First suppose O to be the horizon.

In quadrantal triangle ZOX there are given, OX the observed distance of object from sun, ZX the observed zenith distance, and OZ = 90°, to find the angle OZX.

(2.) In triangle PZS are given, PZ the co-lat., PS the polar distance of sun, and ZS the true zenith dist., to find the true bearing of sun, or the angle PZX.

The sum or difference of the two angles thus found, according to the position of the object with respect to the sun, will be the bearing PZO of object required.

*Calculation.*

(1.) To find angle OZX.

| | |
|---|---|
| app. alt. sun's l. l. | 10° 30′ 0″ |
| index corr. | 0 0 50− |
| | 10 29 10 |
| semi. | 0 15 45 |
| | 10 44 55 |
| dip | 0 3 41 |
| app. alt. sun's center | 10 41 14 |
| cor. for refraction, &c. | 0 4 50 |
| true alt. sun's center | 10 36 24 |

| | |
|---|---|
| ob. dist. n. l | 95° 16′ 0″ |
| in. cor. | 0 1 10+ |
| | 95 17 10 |
| semi. | 0 15 45 |
| OX | 95 32 55 |

In quadrantal triangle ZXO,

cos. OX = sin. ZX . cos. OZX

or − + −

cos. OX . sec. ap. alt. = cos. OZX

| | |
|---|---|
| cos. OX | 8·985491 |
| sec. A alt. | 0·007600 |
| 180° 0′ | |
| 84 21 | 8·993091 |

OZX = 95 39

(2.) To find PZS.

| | |
|---|---|
| lat. | 7° 51′ 0″ |
| tr. alt. | 10 36 24 |
| | 2 45 24 |
| pol. dist. | 112 24 0 |
| | 115 9 24 |
| | 109 38 36 |

| | |
|---|---|
| | 0·004089 |
| | 0·007487 |
| | 4·926411 |
| | 4·912410 |
| | 9·850397 |

∴ PZS 114° 39′ 30″

and OZX 95 39 0

bearing of point . . N. 19 0 30 W.

(2.) In the preceding example suppose O to be the summit of a mountain whose elevation is 10°; it is required to find the true bearing of the point O. In the preceding calculation, to find angle OZX we must substitute the zenith distance (80°) of the point O for the quadrantal arc OZ in last example. We have then three sides given of the triangle OZX, to find as before the angle OZX.

*Calculation.*

| | |
|---|---|
| ZO .. 80° 0′ 0″ | 0·006649 |
| ZX .. 79 18 46 | 0·007594 |
| 0 41 14 | 4·871860 |
| OX .. 95 32 55 | 4·867153 |
| 96 14 9 | 9·753256 |
| 94 51 41 | OZX .... 97° 39′ |
| | and PZS.. 114 39 |

bearing of mountain .. N. 17 0 W.

PROB. 184.

The solution of this problem is given in pp. 47-49.

THE END.

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